Approximate Analytical Study of Thin Film Flow of Third Grade Fluid And Entropy Generation In Pipe

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Abstract

in this study on thin film flow of third grade fluid and entropy generation in pipe , the coupled non-linear ordinary differential equations are solved using perturbation technique. Effects of third grade fluid, magnetic field and Brinkman number on entropy generation number due to heat transfer and fluid friction is investigated. It is observed that increase in the third grade parameter (β) increases entropy generation number more at the region close to the centre of the pipe and less towards the wall of the cylindrical pipe. Influence of magnetic field shows that it has the tendency of enhancing thermodynamic irreversibility which can lead to increased entropy generation. It is further observed that the Brinkman number which is related to the irreversibility of the fluid flow and heat transfer processes increases with fluid viscosity resulting in entropy generation increase, enhances viscous dissipation as such leads to an increase in entropy generation number. However, third grade parameter increases the fluid friction which can lead to a decrease in heat transfer irreversibility resulting in lower Bejan number.

1. Introduction

Third grade fluid is a class of non-Newtonian fluid of the differential type. The non-Newtonian fluid is mathematically complex in computation as a result of it's properties. The constitutive nature of these complex physical model has attracted many instigators. The study of third grade fluid with the assumption of constant viscosity is the simplest model within the Non-Newtonian fluid analysis. Some of the earliest researchers in this field are the Rivlin- Erickson [11], on stress deformation relation for isotropic materials, fosdic and Rajagopal [4] on thermodynamic and stability of fluids of third grade.

Makinde [5] investigated the thermal stability of a reactive third grade fluid in a cylindrical pipe: an exploitation of Hermite-Pade approximation technique. Okoya [9] considered a model for the motion with exponential viscosity of a third grade fluid flowing between parallel plates under the action of externally imposed pressure gradient affects the fully developed and laminar reactive flow. Results were presented for various viscosity variational parameters for which the number is related to the frank-kamenetskii parameter.

Massondi and Christie [6] analyzed the effects of variable viscosity and viscous dissipation of the flow of a third grade fluid in a pipe. Pakdemirli and yilbas [10] studied entropy generation for pipe flow of a third grade fluid with Vogel model viscosity. They introduced Vogel model to account for temperature dependent viscosity. They also formulated flow system for entropy generation due to heat transfer and fluid friction. Results of their analysis shows that increase in the viscosity parameter index A reduces the entropy generation number and increase in the viscosity parameter index B increases entropy generation number.

Obi [8] investigated entropy generation for Non-Newtonian fluid flow with constant viscosity. The governing nonlinear equations of motions were solved analytically using regular perturbation technique, third grade fluid was introduced to account for the non-Newtonian influence. The effects of some physical parameters were examined and results indicate that the third grade parameter has a positive influence on the flow profiles. Results further show that increase in third grade parameter and the Brinkman number increases the temperature of the cylinder, thereby increases entropy generation number.

Bejan [3] examined entropy generation in fundamental convective heat transfer. He showed that engineering design of a thermal system can be improved upon by minimizing the entropy generation. Second law analysis of a swirling flow in a circular duct with restriction was investigated by Yilbas et al [12]. Abu-Hijleh et al [1] studied entropy generation due to natural convection from a horizontal isothermal cylinder in oil.

Obi et al [7] analysed the flow of an incompressible MHD third grade fluid in an inclined rotating cylindrical pipe with isothermal walls and Joule heating. They solved the governing equations and showed that the Eckert and the Grashof parameters reduced the flow velocity. They further observed that the magnetic field parameter, the Grashof number, the Eckert number and the third grade parameter increase the temperature of the cylindrical pipe.

2. Mathematical Formulation

Considering a steady flow of an incompressible MHD third grade fluid through a cylindrical pipe with isothermal wall. Axial pressure gradient was assumed to have induced the fluid motion and both the velocity and the temperature fields depend on r only and the governing non-dimensional equations for the momentum and energy balance can be represented as in Aiyesimi *etal* [2]

$$\frac{d^2u}{dr^2}+6\beta\left(\frac{du}{dr}\right)^2\left(\frac{d^2u}{dr^2}\right)+Mu=-1 \qquad (1)$$

$$\frac{d^2\theta}{dr^2}+B_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+Mu^2=0 \qquad (2)$$

$$\mathrm{u}(0)=0,\ \mathrm{u}(1)=1, \theta(0)=0, \theta(1)=1 \qquad (3)$$

Where u is the velocity of the fluid, θ is the temperature of the cylinder, The terms are related to the non-dimensional variables

$$r=\frac{\overline{r}}{d}, \theta=\frac{T}{T_0}, u=\frac{\overline{u}}{u_0}, \mu=\frac{\overline{\mu}}{\mu_0}, \qquad (4)$$

where r is the radius of the cylinder, T_0 is the reference temperature, u_0 is the reference velocity, μ_0 is the reference viscosity.

$$\mathrm{B}_r=\frac{\mu_0 u_0^2}{kT_0}, \beta=\frac{\beta u_0^2}{\mu_0 d^2} \qquad (5)$$

3. Method of Solution

The semi-analytical solutions for velocity and temperature profiles can be of the form:

$$u(r)=u_0(r)+\beta u_1(r)+\mathrm{O}(\beta^2), \theta(r)=\theta_0(r)+\beta\theta_1(r)+\mathrm{O}(\beta^2),\ \mathrm{M}=\beta\mathrm{M} \qquad (6)$$

Substituting eqn (6) into eqns (1) and (2) and separating each order of β, yields

$$\beta^0: \frac{d^2u_0}{dr^2}=-1 \qquad (7)$$

$$\beta: \ \frac{d^2u_1}{dr^2}+6\left(\frac{du_0}{dr}\right)^2\frac{d^2u_0}{dr^2}+Mu_0=0 \qquad (8)$$

$$\beta^0: \frac{d^2\theta_0}{dr^2}+\mathrm{B}_r\left(\frac{du_0}{dr}\right)^2=0 \qquad (9)$$

$$\beta: \ \frac{d^2\theta_1}{dr^2}+2\mathrm{B}_r\frac{du_0}{dr}\frac{du_1}{dr}+2\left(\frac{du_0}{dr}\right)^4 Mu_0^2=0 \qquad (10)$$

Solving eqns (7-10) with the condition (3), yields

$$u(r)=\frac{1}{2}r-\frac{1}{2}r^2+\beta\left(\frac{3}{4}r^2-r^3+\frac{1}{2}r^4-M\left(\frac{1}{12}r^3-\frac{1}{24}r^4\right)-\frac{1}{4}r+\frac{1}{24}rM\right) \qquad (11)$$

$$\theta(r)=\left(\frac{1}{24}r-\frac{1}{8}r^2+\frac{1}{6}r^3-\frac{1}{12}r^4\right)B_r+\beta\left(-B_r\left(\frac{1}{8}r^3-\frac{1}{4}r^4+\frac{1}{5}r^5\right)-M\left(\frac{1}{96}r^4-\frac{1}{240}r^5\right)\right.$$
$$-\frac{1}{16}r^2+\frac{1}{96}r^2M+M\left(\frac{1}{80}r^5-\frac{1}{180}r^6\right)+\frac{1}{24}r^3-\frac{1}{144}r^3M\Bigg)-\frac{1}{16}r^2+\frac{1}{6}r^3-\frac{1}{4}r^4+\frac{1}{5}r^5$$
$$-\frac{1}{15}r^6-M\left(\frac{1}{48}r^4-\frac{1}{40}r^5+\frac{1}{120}r^6\right)+\left(\left(-\frac{83}{240}r+\frac{1}{120}rM\right)B_r+\frac{57}{240}r-\frac{1}{240}rM\right)\right) \quad (12)$$

4. Viscous Dissipation And Entropy Generation

The viscous dissipation term can be obtained from the equations of motion as:

$$\phi=B_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+Mu^2 \quad (13)$$

The volumetric entropy generation is defined by Bejan (1995) as

$$\Im_{gen}^{'''}=\frac{k}{T_0^2}\left(\frac{d\theta}{dr}\right)^2+\frac{\phi}{T_0} \quad (14)$$

Substituting equation (13) into equation (14), yields

$$\frac{k}{T_0^2}\left(\frac{d\theta}{dr}\right)^2+\frac{1}{T_0}\left(B_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+Mu^2\right) \quad (15)$$

Divide through by $\frac{k}{T_0^2}$, yields

$$\left(\frac{d\theta}{dr}\right)^2+\frac{T_0^2}{kT_0}\left(B_r\left(\frac{du}{dr}\right)^2+2\beta\left(\frac{du}{dr}\right)^4+Mu^2\right) \quad (16)$$

$$\left(\frac{d\theta}{dr}\right)^2+\frac{T_0}{k}\left(\left(\frac{du}{dr}\right)^2\left(B_r+2\beta\left(\frac{du}{dr}\right)^2\right)+Mu^2\right) \quad (17)$$

$$\mathrm{NS}=\left(\frac{d\theta}{dr}\right)^2+\frac{T_0}{k}\left(\left(\frac{du}{dr}\right)^2\left(B_r+2\beta\left(\frac{du}{dr}\right)^2\right)+Mu^2\right) \quad (18)$$

$$\mathrm{NS}_1=\left(\frac{d\theta}{dr}\right)^2 \quad (19)$$

$$\mathrm{NS}_2 = T_0\sigma\left(\left(\frac{du}{dr}\right)^2\left(B_r + 2\beta\left(\frac{du}{dr}\right)^2\right) + Mu^2\right) \quad (20)$$

Where T_0 is the reference temperature, NS is the volumetric entropy generation due to heat transfer and fluid friction, NS_1 is the volumetric entropy generation due to heat transfer and NS_2 is the volumetric entropy generation due to fluid friction.

The ratio

$$\zeta = \frac{NS_1}{NS_2} \quad (21)$$

Is the irreversibility distribution that expresses the heat transfer dominants when $0 < \zeta < 1$ and the fluid friction dominates when $\zeta > 1$. The Bejan number (Be) is defined as

$$Be = \frac{NS_1}{NS} = \frac{1}{1+\zeta}, 0 \le Be \le 1 \quad (22)$$

Consequently, the values of u and θ can be substituted from equations (11) and (12) for final evaluation.

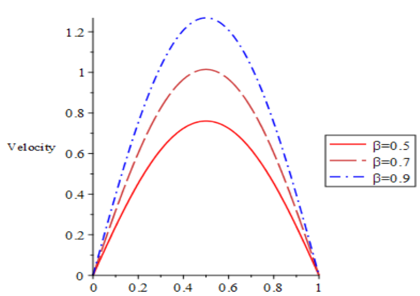

Figure 1: Velocity Profiles For Various Values of Non-Newtonian Parameter(β), $At\ M = 100$

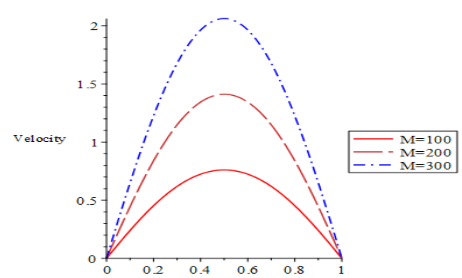

Figure 2: Velocity Profiles For Various Values of Magnetic Field Parameter(M), $At\ \beta = 0.5$

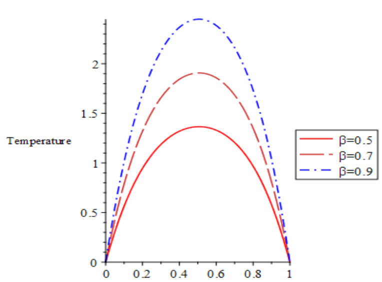

Figure 3: Temperature Profiles For Various Values of Non-Newtonian Parameter(β), $At\ M = 1000,\ B_r = 2.15.$

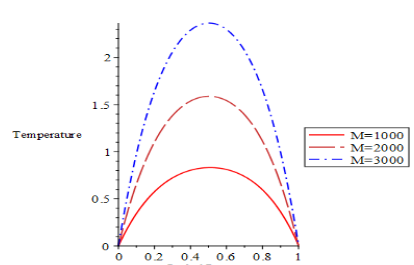

Figure 4: Temperature Profiles For Various Values of Magnetic Field Parameter(M), $At\ \beta = 0.3,\ B_r = 2.05$

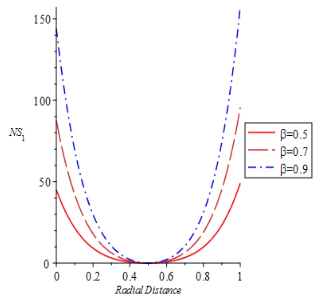

Figure 5: Entropy Generation Number Due To Heat Transfer With Different Values Of Non-Newtonian Parameter (β) At $B_r = 2.15$, $M = 1000$

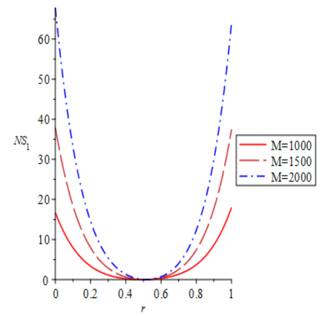

Figure 6: Entropy Generation Number Due To Heat Transfer With Different Values of Magnetic Field Parameter (M) At $\beta = 0.5, B_r = 2.15$

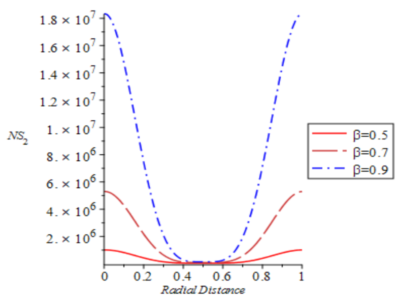

Figure 7: Entropy Generation Number Due To Fluid Friction With Different Values of Non-Newtonian Parameter (β) At $B_r = 0.1$, $T_0 = 7$, $\sigma = 3$, $M = 1000$

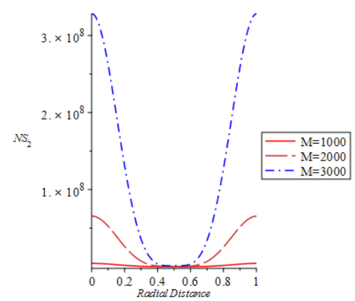

Figure 8: Entropy Generation Number Due To Fluid Friction With Different Values of Magnetic Field Parameter (M) At $\beta = 0.5, B_r = 0.1, T_0 = 7, \sigma = 3$

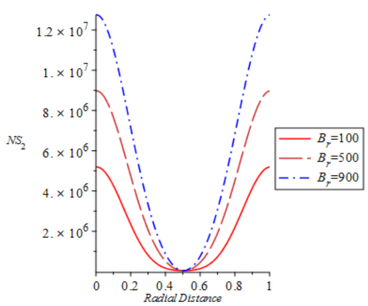

Figure 9: Entropy Generation Number Due To Fluid Friction With Different Values of Brinkmann Number (B_r) At $\beta = 0.5$, $T_0 = 7$, $\sigma = 3$, M=1000

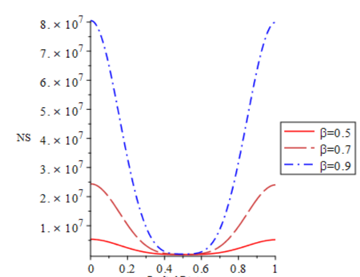

Figure 10: Entropy Generation Number Due To Heat Transfer And Fluid Friction With Different Values of Non-Newtonian Parameter (β) At $B_r = 100, T_0 = 50, \sigma = 15$, M=1000

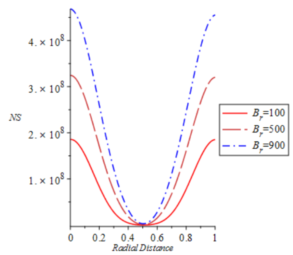

Figure 11: Entropy Generation Number Due To Heat Transfer And Fluid Friction With Different Values of Brinkmann Number (B_r) At $\beta = 0.5$, $B_r = 100$, $T_0 = 50$, $\sigma = 15$, M=1000

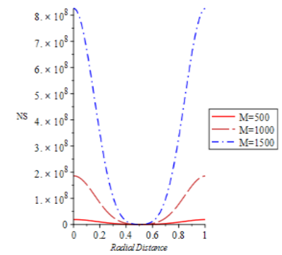

Figure 12: Entropy Generation Number Due To Heat Transfer And Fluid Friction With Different Values of Magnetic Field Parameter (B_r) At $\beta = 0.5$, $B_r = 100$, $T_0 = 50$, $\sigma = 15$.

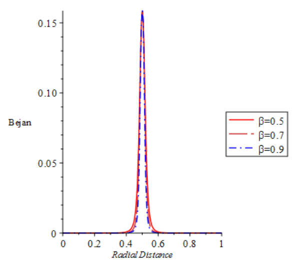

Figure 13: Effects of Non-Newtonian parameter On Bejan Number

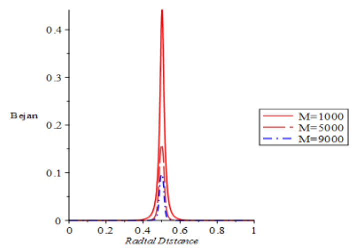

Figure 14: Effects of Magnetic Field Parameter On Bejan Number

5. Results and Discussions

In order to further study the effects of some of the physical parameters involved in the analysis, graphs are presented in figures (1-14). Figures 1 and 3 show the velocity and temperature profiles for different values of the non-Newtonian parameters (β). Results show that increase in non-Newtonian parameter increases the velocity of the fluid flow and the temperature of the cylindrical walls. In figures 2 and 4 are the effects of magnetic field parameter M on the flow velocity and the temperature profiles. It is observed that as the magnetic field parameter increases, the flow velocity and the temperature of the cylinder increases at the same rate. Figure 5 is the entropy generation number due to heat transfer with various values of the non-Newtonian parameter (β) It is clear that increase in the non-Newtonian parameter increases entropy generation number more at the region close to the centre of the pipe and less towards the wall of the cylindrical pipe. Figure 6 is the entropy generation due to heat transfer with different values of the magnetic field parameter (M). It is observed that increase in magnetic field parameter leads to an increase in entropy generation. This is as a result of enhanced Lorentz force which cause disorder in the system. Figure 7 shows the effects of third grade parameter on entropy generation number due to fluid friction. Results indicate that increase in the non-Newtonian parameter, increases entropy generation. This is because shear-dependent viscosity which can lead to increase in entropy generation. It is further seen that entropy generation number is high towards the wall and reduces as radial distance increases towards the centre of the pipe. Figure 8 is the entropy generation number due to fluid friction with various values of magnetic field parameter. Results show that magnetic field has the tendency enhancing thermodynamic irreversibility which can lead to increased entropy generation. Figure 9 shows entropy generation number due to fluid friction for various values of the Brinkman parameter. It is observed that the Brinkman number which is the related to the irreversibility of the fluid flow and heat transfer processes increases with fluid viscosity resulting in entropy generation increase.. Figure 10 shows the effects of non-Newtonian parameter on entropy generation number due to heat transfer and fluid friction on a normal stress condition. Results indicate that entropy generation number increases as the non-Newtonian parameter increases. Figure 11 shows the effects of Brinkman number on entropy generation due to heat transfer and fluid friction. Results show that Brinkman number enhances viscous dissipation as such leads to an increase in entropy generation number. Figure 12 is the entropy generation number due to heat transfer and fluid friction with different values of the magnetic field parameter. It is seen that increase in magnetic field leads to an increase in entropy generation number. Figure 13 shows the effects of non-Newtonian parameter on Bejan number. Results indicate that increase in third grade parameter increases the fluid friction can lead to a decrease in heat transfer irreversibility which also results in lower Bejan number. Figure 14 shows the influence of magnetic field on Bejan number. It is observed that increase in magnetic field leads to a decrease in Bejan number due to the induced electromagnetic forces and increased fluid friction and leads to a decrease in Bejan number.

6. Conclusion

in this study on thin film flow of third grade fluid and entropy generation in pipe , the coupled non-linear ordinary differential equations are solved using perturbation technique. Effects of third grade fluid, magnetic field and Brinkman number on entropy generation number due to heat transfer and fluid friction is investigated. Results show within the constant viscosity that increase in non-Newtonian parameter increases the velocity of the fluid flow and the temperature at the cylindrical walls. It is observed that as the magnetic field parameter increases, the flow velocity and the temperature of the cylinder increases at the same rate. It is seen that increase in the non-Newtonian parameter (β) increases entropy generation number more at the region close to the centre of the pipe and less towards the wall of the cylindrical pipe. Influence of magnetic field shows that it has the tendency of enhancing thermodynamic irreversibility which can lead to increased entropy generation. It is observed that the Brinkman number which is related to the irreversibility of the fluid flow and heat transfer processes increases with fluid viscosity resulting in entropy generation increase, enhances viscous dissipation as such leads to an increase in entropy generation number. However, third grade parameter increases the fluid friction which can lead to a decrease in heat transfer irreversibility resulting in lower Bejan number.

Declaration

1. Funding: Not applicable
2. Informed Consent Statement: Not applicable
3. Data Availability: Not applicable
4. Conflict of Interest: No conflict of interest

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