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"Numerical Computation of Reactive Flow of Third Grade Fluid With Heat Generation"

Numerical Computation of Reactive Flow of Third Grade Fluid With Heat Generation

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Abstract

Computation of reactive flow of third grade fluid in cylindrical pipe with heat generation is considered. The resulting governing equations of motion are highly nonlinear and are solved using collocation method in verifying the impacts of some material variables involved. Results indicate that increase in the non-Newtonian variable increases the flow velocity and decreases the temperature of the walls of the cylindrical pipe. It is observed that the critical Frank-Kamenetskii variable exist for which the solution fails to be distinct. It is further observed that for $\xi > \xi_c$, a steady state solution does not exist suggesting a thermal runaway which could be avoided by setting $\xi = 1.8879565$.

Keywords: Numerical, third grade, heat generation, reactive, computation.

1. Introduction

Third grade fluid is in the class of non- Newtonian fluids of the differential type. It is a class of non- Newtonian fluid where the stress tensor is the addition of all the tensors that can be developed from the velocity field with up to three derivatives. Example of this class of fluid include ketchup, paints, blood etc. chemical reactions involved in fluid which is capable of altering the fluids properties and composition in flow process is referred to as reactive flow. There are some studies now available in literature. Some of the earliest work on third grade fluid are Fosdick and Rajagopal [3] analyzed the thermodynamic third grade fluid and showed the restrictions on the stress constitutive model. Rajagopal [10] examined the stability properties of third grade fluids. Szeri and Rajagopal [13] investigated the flow of third grade fluids between heated parallel plates. Ellahi et al [1] examined the impacts of slip on the nonlinear flows of a third grade fluid. In the investigation, the results of no-slip condition were inferred as a restricting case when the slip variable is equal to zero.

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Makinde [4] investigated a steady flow of a reactive variable viscosity fluid in a cylindrical pipe with an isothermal wall. Makinde [5] studied the thermal criticality for reactive gravity-driven thin film flow of a third grade fluid with adiabatic free surface down an inclined plane. Salawuland Fatumbi [11] examination the inherent irreversibility of hydromagnetic third grade reactive poiseuille flow with variable viscosity. The employed the weighted residual method for the solution. The study shows that the heat dissipation of reactive exothermic chemical in a uniform magnetic field moved past fluid in a porous medium in an irreversible mode. The study on steady flow of a reactive viscous fluid in porous cylindrical pipe was carried out byFarayola [2]. The investigation was done using regular perturbation for the solution of the nonlinear equation. Okedayo et al [9] numerically investigated the reactive MHD flow of thrd grade fluid in pipe.The study involved the weighted residual collocation method which was use as a computational means of solution and were able to display the influence of various thermo-physical parameter. It was observed that the critical value of the Frank-Kamenetskii and third grade parameters exists for which the solution sizes to be unique. Obi et al [7] analyzed the incompressible flow of third grade fluid in an inclined rotating cylindrical pipe with isothermal wall and Joule heating. They solved the nonlinear equations by perturbation method and the effect of some parameter on the flow presented graphically. Yurusoyand Pakdemirli [14] investigated the fluid flow of third grade fluid in pipe with heat transfer with a case of constant viscosity. Reynold'sand Vogel's models were introduced to account for temperature-dependent viscosity. They analytically examined the flow and compared the result with the finite difference procedure earlier given by Massaudi and Christie [6]. Okedayo et al [8] focuses on Gaterkinweighted residual method for magnetohydrodynamic mixed convection flow in a vertical channel filled with porous media. Results obtained were analyzed using tables and graphs. Siddiqui et al [12] analyzed the thin film flow of third grade fluid down an inclined plane using the combined traditional perturbation as well as the homotopy perturbation methods and comparison made of the two results obtained from the two techniques.

This research seeks to examine the consequences of the Frank-Kamenetskii parameter on the reactive flow of third grade fluid in cylindrical pipe.

2. Formulation of the Problem

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The fundamental models of an incompressible viscous flow are the mass and momentum conservation laws which in the absence of heat transfer process, such laws are defined through continuity and momentum equations.

In vector form,

$$\nabla . u = 0 \tag{1}$$

$$\rho \frac{Du}{Dt} = -\nabla \rho + p\mathrm{f} + \mathrm{div}\,\tau - \frac{\mu \phi u}{k} \tag{2}$$

where

ρ =density, u =velocity, p =pressure, τ =stress tensor, f = body force and

$\frac{D}{Dt}$ =material derivative

Stress tensor defining a third grade fluid is given by

$$\tau = \sum_{i=1}^{3} \mathrm{B}_i \tag{3}$$

where

$$\mathrm{B}_1 = \mu A_1$$

$$\mathrm{B}_2 = \alpha_1 A_1 + \alpha_2 A_1^2$$

$$\mathrm{B}_2 = \beta_1 A_2 + \beta_2 \left(\mathrm{tr} A_1^2 \right) A_1 \tag{4}$$

μ is the coefficient of dynamic viscosity, $\alpha_1, \alpha_1, \beta_1, \beta_1$ are constants.

The Rivlin-Ericksen tensors A_n are defined by $A_0 = 1$, being the identity tensor.

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1} \left(\nabla u \right) + \left(\nabla u \right)^t A_{n-1}, \; n \geq 1 \tag{5}$$

Assuming the surface tension to be negligible, we have the velocity field in t he

$$U = \left(u(r), 0, 0 \right) \tag{6}$$

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Substituting the values of $\nabla.\tau$ and v in equation (2), yields

$$\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right)+\frac{\beta\mu}{k}u-\sigma B_0^2u=\frac{dp}{dz} \tag{7}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\mu}{k}\left(\frac{du}{dr}\right)^2+\sigma B_0u^2+QC_0A\exp\left(-\frac{E}{RT}\right)=0 \tag{8}$$

$$\frac{du}{dr}(0)=\frac{dT}{dr}(0)=0,u(a)=0,T(a)=T_0 \tag{9}$$

where

u is fluid velocity, T is absolute temperature, μ is dynamic viscosity, σ ius electricalconductivity, T_0 is reference temperature, a is radius of the pipe, r is radial distance, E is the activation energy, R is the universal gas constant, C_0 is concentration, Q is heat activationand A is the rate constant. Introducing the following dimensionless parameters:

$$r=a\bar{r},u=u_0\bar{u},\theta=\frac{E}{RT_0}(T-T_0),\in=\frac{RT_0}{E},\lambda=\frac{Ea^2QC_0A\ell^{-\frac{E}{RT_0}}}{RT_0^2k} \tag{10}$$

Using eqn (10) in eqns (7-9), yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\delta\beta u-Mu=-1 \tag{11}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)+B_r\left(\frac{du}{dr}\right)^2+Mu^2+\tau e^{\frac{\theta}{\in\theta+1}}=0 \tag{12}$$

$$\frac{du}{dr}(0)=\frac{d\theta}{dr}(0)=0,u(1)=0,\theta(0)=0 \tag{13}$$

3. Method of Solution

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In order to solve the nonlinear momentum and energy equation of (11) and (12) with the condition (13), we employ collocation method for the solution. This technique is a function approximation method which reduces the nonlinear ordinary differential equations to algebraic equations which can be solved byany iterative fixed point technique.

The approximate solution is of the form

$$u(r)=\sum_{j=0}^{n} a_j \phi_j(r) \tag{14}$$

Equation (14) is the trial function over the region which must satisfy the given boundary conditions. In this technique, the one, two and three term coefficients are employed and the maximum velocity and temperature ascertained.

$$u_0(r)=a_0(1-r^3), u_1(r)=a_0(1-r^3)+a_1(r^2-r^3) \tag{15}$$

Similarly,

$$\theta_0(r)=a_2(1-r^3), \theta_1(r)=a_2(1-r^3)+a_3(r^2-r^3) \tag{16}$$

$$-4.5000000000\theta_{\max}+0.05321635176+\zeta\frac{0.875\theta_{\max}}{e^{1+0.000875\theta_{\max}}} \tag{17}$$

$$a_0=0.2222222222 \tag{18}$$

The thermalcritical property is determined bythe relationship between maximum temperature and the Frank-Kamenetskii variable and achieved from equations (17)and (18) with other thermo-solutal parameters. Plotting equation (17) results in figure 5.

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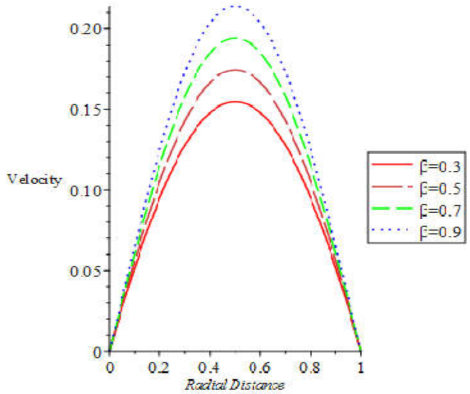

Figure 1: Velocity Profiles For Various Values of The Third Grade Parameter (β)

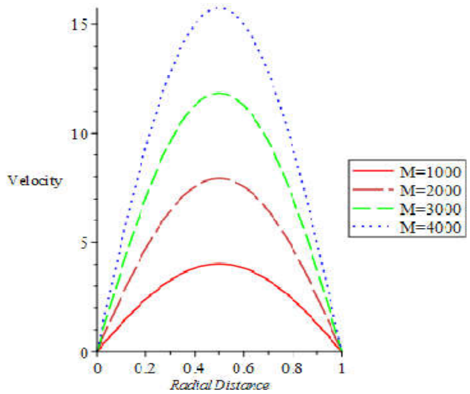

Figure 2: Velocity Profiles For Various Values of The Magnetic Field Parameter (M)

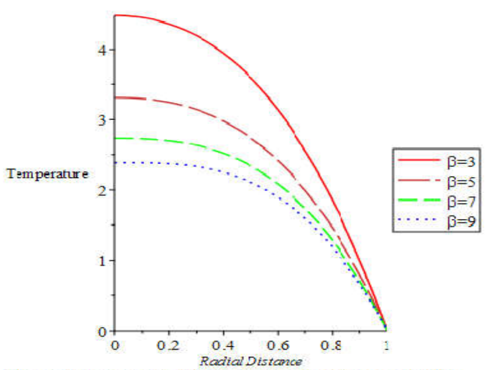

Figure 3: Temperature Profiles For Various Values of Third Grade Parameter (β)

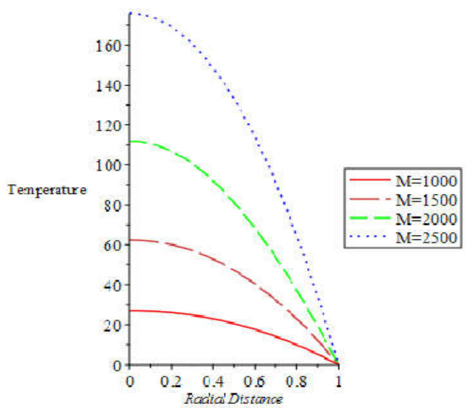

Figure 4: Temperature Profiles For Various Values of Magnetic Field Parameter (M)

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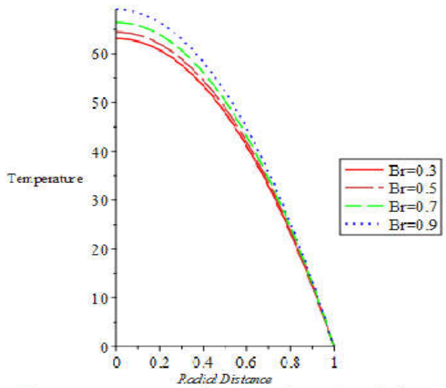

Figure 5: Temperature Profiles For Various Values of The Brinkmann Parameter (Br)

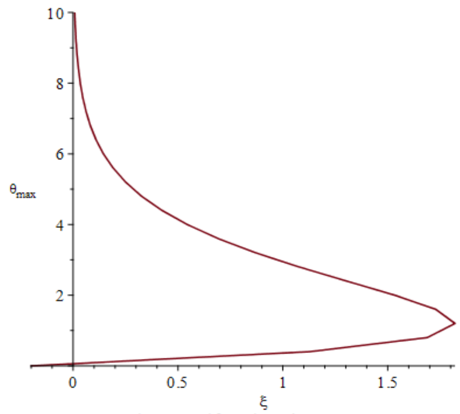

Figure 5: Bifurcation Chart

4. Results and Discussions

In this section, the impacts of some variables that are of great importance to the study are discussed. These effects come from the graphs presented in earlier section. In figures 1 and 2, the influence of non-Newtonian and magnetic field variables are respectively shown. It is seen from the results that increase in both parameters enhance the flow velocity. Figure 3 shows the temperature profiles for values of the non-Newtonian variable. Result shows that increase in the parameter β, decreases the temperature of the cylindrical pipe at the walls. Figure 4 is the temperature profiles for various values of the magnetic field parameter. Results indicate that increase in the magnetic field parameter increases the temperature of the system. This is because magnetic field can generate heat through electrical resistance, thereby leads to a rise in temperature. Figure 5 shows the critical value of the Frank-Kamenetskii parameter $\xi_c = 1.8879565$.

5. Conclusion

Computation of reactive flow of third grade fluid in cylindrical pipe with heat generation is considered. The resulting governing equations of motion are highly nonlinear and are solved using collocation method in verifying the impacts of some material variables involved. Results indicate that increase in the non-Newtonian variable increases the flow velocity and decreases the temperature of walls of the cylindrical pipe. it is observed that the critical

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Declarations

1. Funding: Not applicable
2. Informed Consent Statement: Not applicable
3. Data Availability: Not applicable
4. Conflict of Interest Statement: No conflict of interest

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