IJO - International Journal of Mathematics

(ISSN 2992-4421) *EGBEYEMI, H. B *

https://ijojournals.com/ Volume 09 || Issue 07 || July, 2026 ||

"Thermal Analysis of the Effect of Non-Linear Thermal Radiation and Non-Uniform Internal Heat Generation on a MHD Combustible Third-Grade Fluid Flowing Through a Vertically Positioned Darcy Porous Channel"

Thermal Analysis of the Effect of Non-Linear Thermal Radiation and Non-Uniform Internal Heat Generation on a MHD Combustible Third-Grade Fluid Flowing Through a Vertically Positioned Darcy Porous Channel

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Abstract

This study presents the thermal analysis of a magnetohydrodynamic combustible third-grade fluid flowing through a Darcy porous channel that is vertically positioned and subjected to the combined effect of Darcy Forchheimer inertia, non-linear thermal radiation and non-uniform internal heat generation/absorption. The fluid layer in contact with the channel walls is subjected to asymmetric slip conditions and constant temperature boundaries. The coupled non-linear system of ordinary differential equations arising from the phenomena are subjected to the procedure of nondimensionalization. The resulting dimensionless equations are then solved numerically by Two-Point Taylor Polynomial Iterative Linearization Technique with Legendre nodes (TTPILT). The methodology associated with the numerical approach begins with the linearization of non-linear terms in the governing equations, followed by the application of the Legendre nodes collocation technique to iteratively obtain solutions. The iterative process continues until convergence to the desired solution is achieved. The accuracy of the method are validated by comparing the obtained results with those generated using the spectral Chebyshev collocation method, which shows good agreement. Graphical illustrations of the fluid velocity and temperature profiles are provided and discussed. The findings on flow characteristics and thermal behavior have significant implications for the design and optimization of thermodynamic systems. In particular, the thermal analysis offers valuable insights for developing energy-efficient systems with optimal performance.

Keywords: Combustible third-grade fluid, variable viscosity, non-linear thermal radiation, non-uniform internal heat generation.

1. Introduction

A great deal of focus has been given to the study of combustible fluids by researchers due to its vital role in a wide range of technological and industrial applications, ranging from power plants to internal combustion engines. As modern systems grow more reliant on complex chemical processes, whether in energy production, pharmaceuticals, combustion systems, or chemical manufacturing, understanding how combustible fluids behave has become essential. These flows are shaped not only by traditional fluid mechanics but also by the chemical reactions occurring within them, which can dramatically alter temperature, pressure, and flow characteristics [1-3]. Combustible fluids have improved the thermal performance of engineering devices by providing efficient energy through controlled combustion. Fuels like gasoline, diesel, and natural gas produce large amounts of heat that can be converted into mechanical work or power. This enhances the efficiency of systems such as internal combustion engines, gas turbines, and boilers, making them more powerful, compact, and effective in applications like transportation, power generation, and industrial processes [4-6].

The operation of many engineering and technological systems relies heavily on the process of combustion, which involves the transport and ignition of combustible fluids. These fluids, characterized by their high flammability, can ignite and burn easily, often presenting significant fire hazards. Extensive research has been conducted on the behavior and dynamics of combustible fluids, with several notable studies highlighted and discussed as follows. Ohaegbue *et al.,* [7] analyzed heat dissipation in a two-step reacting tangent hyperbolic fluid using a quadratic Boussinesq approximation with convective cooling, focusing on how internal heat generation affects combustion stability. By solving the governing equations numerically through the Galerkin weighted residual method, they found that parameters such as the Frank-Kamenetskii parameter, Brinkman number, Weissenberg number, activation energy, and reaction-related terms significantly enhance combustion, but also increase the risk of thermal runaway if not carefully controlled. In contrast, Chang *et al.,* [8] approached combustion from a safety perspective, examining how temperature, pressure, and inert gases influence the flammability limits of combustible gases; their study highlights the elevated explosion risks in industrial environments operating under extreme conditions. Similarly concerned with combustion dynamics but at a more fundamental level, Mikhali *et al.,* [9] investigated heat flux and ignition delay in shock-heated gases, stressing the importance of accurate ignition modeling for high-speed aerospace applications. Meanwhile, Alexander *et al.,* [10] focused on flame propagation behavior, using three-dimensional simulations to study downward flame spread over solid combustible materials and validating their results experimentally. In comparison, Ohaegbue *et al.,* emphasize internal heat generation and parameter sensitivity in non-Newtonian reactive fluids, whereas Chang *et al.,* concentrate on external safety limits and explosion risks under varying environmental conditions. Mikhali et al. [9] provide insight into ignition timing and combustion initiation, which complements both studies by addressing early-stage combustion behavior, while Alexander *et al.,* extend the discussion to flame spread and material interaction. Together, these works offer a comprehensive perspective, spanning combustion enhancement, safety considerations, ignition characteristics, and flame propagation in different physical and engineering contexts.

The study of fluid movement whether Newtonian, non-Newtonian, or third-grade under the influence of magnetic fields is a significant area in fluid mechanics. This is because magnetic fields can be used to control or stabilize fluid flow and suppress turbulence within channels. The interaction between magnetic fields and electrically conducting fluids, known as magnetohydrodynamics (MHD), has attracted considerable attention from scientists and researchers over the years. This growing interest is driven by MHD's wide range of important applications across various fields, including physics, engineering, and technology, among others [11]. Magnetohydrodynamics (MHD) has found numerous applications across various fields, including materials processing such as electromagnetic casting and crystal growth, fusion research through devices like stellarators, and astrophysics, encompassing stellar dynamics, the solar wind, and cosmic evolution [12]. Over the years, researchers have made significant contributions toward advancing the understanding of MHD. Some of these key developments are discussed hereafter. Farooq *et al.,* [13] developed a mathematical model describing electromagnetohydrodynamic multiphase flows incorporating nano-sized particles, with the practical goal of improving diffuser-type injector performance in modern automotive engines. Their work focused on synthesizing complex physical effects, electric fields, magnetic fields, and multiphase nano-fluid behavior, into a unified model capable of guiding engineering design. The study emphasized the relevance of such advanced fluid models in enhancing efficiency and operational stability in high-performance automotive systems. In contrast, Jamil *et al.,* [14] concentrated on validating numerical approaches for modeling blood flow under magnetohydrodynamic conditions with hybrid nanofluids. By comparing the Akbari–Ganji Method (AGM), the fourth-order Runge–Kutta Method (RKM), and the Finite Volume Method (FVM), the authors demonstrated that no single technique is universally superior; instead, suitability depends on problem complexity, required accuracy, and computational limitations. Both studies highlight the importance of robust mathematical and numerical tools for understanding complex MHD-based fluid systems. While Farooq *et al.,* apply modeling to engineering design, Jamil *et al.,* evaluate computational strategies, collectively underscoring that precise modeling and method selection are critical for advancing applications involving magnetic fields, nanofluids, and intricate flow behavior.

Non-uniform internal heat generation means heat is produced at different rates across a material instead of evenly. In porous channels carrying combustible fluids, this happens due to chemical reactions and variations in the medium, creating localized hot and cool regions. These temperature differences strongly affect fluid behavior, changing viscosity, flow speed, and reaction rates. Hotspots can accelerate reactions and even cause instability or thermal runaway, while cooler areas slow things down. The overall process involves a complex interaction of heat transfer, fluid flow, and chemical reactions. This concept is important in systems like catalytic and packed bed reactors, porous burners, fuel cells, and enhanced oil recovery, where controlling heat distribution is key to safety and efficiency [15]. Oyeyinka *et al.,* [16] investigated magnetohydrodynamic (MHD) flow of a non-Newtonian Williamson fluid over an inclined stretching sheet embedded in a porous medium, incorporating mixed convection and non-uniform internal heat generation. Using similarity transformations, the governing partial differential equations were reduced to ordinary differential equations and solved semi-analytically via Legendre polynomial-based Gauss–Lobatto collocation, with validation from the shooting Runge–Kutta method. Their results show that buoyancy effects (Grashof number) enhance fluid velocity, while increased porous resistance suppresses flow but elevates temperature levels. Additionally, spatially varying heat generation produces steep thermal gradients that induce localized thermal stresses, highlighting the sensitivity of such systems to permeability and heat source variations. On the other hand, Alao *et al.,* [17] focused on MHD mixed convection of nanofluids in a stretching/shrinking microchannel using the Tiwari–Das model, considering Silver and Iron Oxide nanoparticles with non-uniform heat generation or absorption. The transformed ODE system was solved numerically using a fourth-order Runge–Kutta shooting technique. Their findings reveal that nanoparticle inclusion significantly enhances heat transfer and near-wall velocity, with Iron Oxide nanofluids showing stronger thermal accumulation and thicker boundary layers. While both studies agree that increasing internal heat generation strengthens fluid motion through buoyancy effects, they differ in emphasis: Oyeyinka *et al.,* highlight the restrictive role of porous media and nonlinear fluid behavior, whereas Alao *et al.,* demonstrate how nanoparticle addition enhances thermal performance and flow characteristics, making their study more directly applicable to advanced thermal engineering systems.

The Darcy–Forchheimer law is highly relevant for describing the flow of a combustible fluid through a porous-filled vertical channel because it extends classical Darcy's law by incorporating inertial effects that become significant at higher velocities, which are common in combustion-driven or buoyancy-assisted flows. In such systems, the presence of a porous matrix alters velocity distribution, heat transfer, and species diffusion, while the vertical orientation introduces natural convection due to temperature gradients from combustion. The Forchheimer term accounts for nonlinear drag, enabling more accurate prediction of pressure drop and flow stability, both of which are critical in preventing flashback or flame instability. This modeling approach is particularly important in engineering and industrial applications such as packed-bed reactors, thermal insulation systems, catalytic combustion units, filtration of flammable gases, and enhanced oil recovery, where controlling heat and mass transfer in reactive porous structures directly impacts efficiency, safety, and performance [18]. The study by Soni *et al.*, [19] provides an experimental investigation into non-Darcy flow behavior in porous materials, highlighting the limitations of traditional Reynolds number-based approaches for predicting inertial effects. Focusing on cryogenic dry-shippers, the authors showed that rapid nitrogen evaporation leads to high gas velocities, making it necessary to incorporate both the Klinkenberg effect and the Darcy–Forchheimer law for accurate modeling. Their results revealed that even when Reynolds number criteria suggest otherwise, inertial effects remained significant, especially in materials with complex pore geometries. By experimentally determining permeability and related parameters through pressure-drop analysis and numerical integration, they establish that the Forchheimer number is a more reliable indicator of form drag, emphasizing the need for improved characterization techniques in porous material design. Also, Raza *et al.*, [20] and Hamid *et al.*, [21] adopted computational approaches to explore heat and mass transfer in advanced fluid systems within Darcy–Forchheimer porous media. Raza *et al.*, examined ternary hybrid ferrofluids under magnetohydrodynamic conditions, demonstrating enhanced thermal performance and showing how

IJO - International Journal of Mathematics

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https://ijojournals.com/ Volume 09 || Issue 07 || July, 2026 ||

parameters like activation energy can reduce heat and mass transfer rates, while other factors improved thermal efficiency. Similarly, Hamid *et al.*, analyzed bioconvective nanofluid flow with additional effects such as thermal radiation and thermophoresis, finding that these factors increased temperature while the Forchheimer parameter reduced wall shear stress. While Panel *et al.*, focused on experimental validation and challenge conventional flow criteria, the latter studies emphasized numerical modeling and optimization of transport phenomena. Together, they illustrate how the Darcy–Forchheimer framework can be applied across both practical material evaluation and advanced thermal-fluid system design.

Nonlinear thermal radiation becomes important in the flow of combustible fluids through a porous channel when temperature differences are large enough that radiative heat transfer no longer scales linearly with temperature, typically following a T^4dependence as described by the Stefan–Boltzmann law. In such systems, the porous medium influences both fluid resistance and heat distribution, while the combustible nature of the fluid introduces strong coupling between temperature, reaction rates, and flow stability. Nonlinear radiation can significantly alter temperature profiles, enhancing or damping thermal gradients, which in turn affects ignition conditions, flame propagation, and the risk of thermal runaway. This is particularly relevant in high-temperature environments where conduction and convection alone cannot accurately describe heat transfer. A practical engineering application is found in packed-bed reactors used in chemical and petrochemical industries, where combustible gases flow through porous catalytic beds. In this scenario, nonlinear radiation impacts reactor efficiency, hotspot formation, and safety [22]. Ibiesam *et al.*, [23] explored the influence of thermal jump on hybrid nanomaterials subjected to nonlinear thermal radiation within a porous medium. The authors highlighted the wide-ranging applications of hybrid nanomaterials in engineering and biomedical systems, particularly emphasizing the relevance of magnetohydrodynamic (MHD) nanomaterials in medical diagnostics and treatment processes. Their study focused on peristaltic transport of a hybrid nanofluid containing iron oxide (Fe_3O_4) and gold (Au) nanoparticles, incorporating electroosmotic effects modeled through the Nernst–Planck and Poisson equations. The formulation accounted for Lorentz force, Darcy–Forchheimer porous resistance, mixed convection, Joule heating, viscous dissipation, and heat generation, while adopting a corrugated symmetric channel. Under the assumptions of long wavelength and low Reynolds number, the governing equations were simplified via lubrication theory and solved numerically. The results demonstrated that Darcy and Forchheimer parameters exert opposite influences on velocity, while thermal jump enhances temperature distribution, and hybrid nanomaterials exhibit superior heat transfer performance. In a related but distinct framework, Gangadhar *et al.*, [24] analyzed Maxwell viscoelastic fluid flow between spiraling disks, incorporating nonlinear thermal radiation alongside homogeneous and heterogeneous chemical reactions. Using the Cattaneo–Christov heat flux model to capture thermal relaxation, the governing equations were transformed via von Kármán similarity variables and solved numerically, with further validation using regularized machine learning techniques. Their findings revealed that increasing Deborah number reduces velocity and pressure, while stronger chemical reaction parameters decrease concentration. Additionally, the regularized linear regression model outperformed other predictive methods. Similarly, Gowda *et al.*, [25] investigated nanofluid flow between two parallel disks under a high-oscillating magnetic field, also employing the Cattaneo–Christov heat flux model and incorporating thermal radiation and chemical reactions. The system assumed a rotating upper disk and an expanding lower disk, and the transformed equations were solved using the Touchard Polynomial Collocation Technique (TPCMT), with validation against the Runge–Kutta–Fehlberg (RKF-45) method. Their results indicated that increasing the rotation parameter reduces axial velocity, while higher temperature relaxation diminishes the thermal field, in contrast to radiation, which enhances temperature. In comparison, all three studies examined advanced heat transfer in non-Newtonian or nanofluid systems under nonlinear thermal radiation, but differ significantly in physical configuration and modeling approach. Ibiesam *et al.*, focused on peristaltic hybrid nanofluid flow with electroosmosis and porous media effects, making their work more applicable to biomedical and microfluidic systems. In contrast, Gangadhar *et al.*, and Gowda *et al.*, considered rotating disk geometries with chemical reactions and thermal relaxation, aligning more with industrial and chemical processing applications. Methodologically, Ibiesam *et al.*, relied on lubrication-based numerical solutions, Gangadhar *et al.*, uniquely integrated machine learning validation, and Gowda *et al.*, applied a collocation technique with numerical benchmarking. Despite these differences, a common outcome across the studies is that key parameters—such as

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(ISSN 2992-4421) *EGBEYEMI, H. B *

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magnetic effects, thermal radiation, and reaction rates, strongly regulated velocity, temperature, and concentration fields, thereby aiding the underscoring their importance in optimizing heat and mass transfer processes.

This present work is set out to study the thermal behavior of an MHD combustible third-grade fluid in a Darcy porous vertical channel with nonlinear radiation and nonuniform internal heat generation because it provides a realistic model of heat transfer in complex systems. Also, third-grade fluids capture non-Newtonian effects, while nonlinear radiation and internal heat generation account for high-temperature and reactive conditions. Including MHD and porous media allows control of flow and represents practical environments. This analysis is essential for predicting overheating, improving heat transfer, and ensuring safety, with applications in reactors, energy systems, polymer processing, and geothermal technologies.

2. Model Formulation

This work examines the steady flow of an incompressible, temperature dependent viscous combustible third-grade fluid through a porous medium filled vertical channel that is influenced by Darcy Forchheimer inertia, non-linear thermal radiation and non-uniform internal heat generation. The flow is governed by a one-step reactive magnetohydrodynamic (MHD) process, with constant thermal and electrical conductivities. The walls of the channel are assumed to experience fluid slippage. It is also assumed that the flow is induced by an applied axial constant pressure gradient and buoyancy force. Emanating from the work of the authors in [26-29] and neglecting fluid consumption, the non-dimensionless model equation that mirrors the flow which are the momentum equation and the energy equation can be written as,

Momentum equation

$$\begin{aligned}&\frac{d}{dy'}\left(\bar{\mu}(J)\frac{du'}{dy'}\right)-\rho V_o\frac{du'}{dy'}-\frac{dP'}{dx'}+6\beta_3\left(\frac{du'}{dy'}\right)^2\frac{d^2u'}{dy'^2}-\frac{\bar{\mu}(J)}{K}u'-\beta^*\rho u'^2\\&-\sigma_o B_0^2u'+\rho g\beta(J-J_o)=0\end{aligned}\tag{1}$$

Energy equation

$$\begin{aligned}&\kappa\frac{d^2J}{dy'^2}-\rho C_p V_o\frac{dJ}{dy'}+\left(\frac{du'}{dy'}\right)^2\left[\bar{\mu}(J)+2\beta_3\left(\frac{du'}{dy'}\right)^2\right]+\frac{\bar{\mu}(J)}{K}u'^2+\beta^*\rho u'^3\\&+\sigma_o B_o^2u'^2+QC_oA\left(\frac{KJ}{\upsilon l}\right)^m e^{-\frac{E}{RJ}}+\frac{16\sigma^*}{3K^*}\left(J^3\frac{d^2J}{dy'^2}+3J^2\left(\frac{dJ}{dy'}\right)^2\right)+\left[A^*(J-J_o)+B^*\frac{RJ_o^2}{E}F'\right]=0\end{aligned}\tag{2}$$

Subject to the appropriate boundary condition

$$\left.\begin{aligned}&y'=-a,\ \lambda_1u'=\bar{\mu}(J)\frac{du'}{dy'}+2\beta_3\left(\frac{du'}{dy'}\right)^3, J=J_o\\&y'=a,\ \lambda_2u'=-\bar{\mu}(J)\frac{du'}{dy'}+2\beta_3\left(\frac{du'}{dy'}\right)^3, J=J_o\end{aligned}\right\}.\tag{3}$$

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The modified fluid pressure is denoted as P', x' and y' are the axial and normal coordinates to the vertically positioned channel, the fluid velocity is represented as u', the heat transfer coefficient at the lower plate is denoted as h_1, The initial temperature of the fluid is given as J_o, J is the fluid temperature, the acceleration due to gravity is represented as g, the third-grade

material coefficient is denoted as β_3,the porous medium permeability is represented as K, μ is the fluid dynamic viscosity, the fluid density is denoted as ρ, the heat generated internally is denoted as Q, the initial concentration of the reactant species is denoted as C_o, the reaction rate constant is denoted as A, the slip coefficients at the lower and upper walls of the channel are represented as λ_1 and λ_2 respectively, the activation energy is represented as E, the Boltmann's constant is denoted as h, l is the Planck's number, the universal gas constant is represented as R, the vibration frequency is denoted as υ, the volumetric thermal expansion coefficient is represented as β, β^* is the Forchheimer inertia coefficient, m is the numerical exponent such that the three values representing numerical exponents for Arrhenius, sensitized and biomolecular kinetics are given respectively as $m \in [0.5, 0, -2]$ [30-31]. The temperature dependent viscosity is written as

$$\overline{\mu}(J) = \mu_o e^{-b(J-J_o)}. \tag{4}$$

Where μ_o is the initial fluid viscosity at temperature J_o and b is the viscosity variation parameter.

By implementing the following non-dimensional variables in equation (5) to equations (1) – (3),

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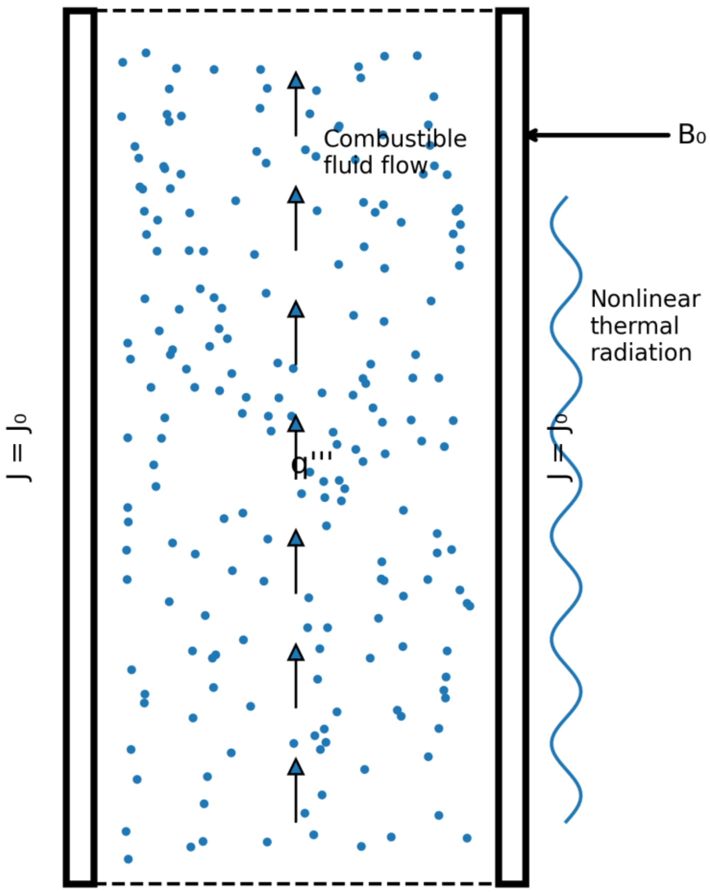

Figure 1: Schematic diagram of the flow

$$y = \frac{y'}{a}, x = \frac{x'}{a}, F = \frac{pau'}{\mu_o}, \theta = \frac{E(J - J_0)}{RJ_o^{\ 2}}, P = \frac{a^2 \rho P'}{\mu_o^{\ 2}}. \tag{5}$$

the dimensionless governing equations are obtained

Dimensionless Momentum Equation

$$0 = \left(e^{-\alpha\theta} F'' - \alpha e^{-\alpha\theta} F'\theta'\right) - \operatorname{Re} F' + 6\Omega F''\left(F'\right)^2 - \delta e^{-\alpha\theta} F - F^{\Delta} F^2 - MF + Gr\theta\sin(\omega) + G. \tag{6}$$

Where

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$$\left.\begin{array}{l}\mathrm{Re}=\dfrac{\rho aV_o}{\mu_o},\ \alpha=\dfrac{bRT_o^{\,2}}{E},\ G=-\dfrac{dP}{dx},\\ \Omega=\dfrac{\beta_3\mu_o}{\rho^2a^4},\ \delta=\dfrac{a^2}{K},\ M=\dfrac{\sigma_oB_o^{\,2}a^2}{\mu_o},\ Gr=\dfrac{\rho^2g\beta a^3RT_o^{\,2}}{\mu_o^{\,2}E},\ F^{\Delta}=\beta^*a\end{array}\right\}. \tag{7}$$

Dimensionless Energy Equation

$$\begin{array}{l}0=\theta''-\mathrm{Re}\,\mathrm{Pr}\,\theta'+\lambda\left[\left(1+\varepsilon\theta\right)^m e^{\frac{\theta}{1+\varepsilon\theta}}+\gamma\left\{\left(F'\right)^2\left(e^{-\alpha\theta}+2\Omega\left(F'\right)^2\right)+\delta e^{-\alpha\theta}F^2+F^{\Delta}F^3+MF^2\right\}\right]\\ +\left[\Lambda_1\theta+\Lambda_2F'\right]+Nd\left[(1+\varepsilon\theta)^3\theta''+3\varepsilon(1+\varepsilon\theta)^2\theta'^2\right]\end{array} \tag{8}$$

Where

$$\left.\begin{array}{l}\alpha=\dfrac{bRJ_o^{\,2}}{E},\ \varepsilon=\dfrac{RJ_o}{E},\ \Omega=\dfrac{\beta_3\mu_o}{\rho^2a^4},\ \ \delta=\dfrac{a^2}{K},\\ \lambda=\dfrac{QC_oAa^2E}{\kappa RJ_o^{\,2}}\left(\dfrac{KJ_o}{\upsilon l}\right)^m e^{-\frac{E}{RJ_o}},\ \gamma=\dfrac{\mu_o^{\,3}}{QC_oA\rho^2a^4}\left(\dfrac{\upsilon l}{KJ_o}\right)^m e^{\frac{E}{RJ_o}}\\ Nd=\dfrac{16\sigma^*J_o^{\,3}}{3\kappa K^*},\ M=\dfrac{\sigma_oB_o^{\,2}a^2}{\mu_o},\ \mathrm{Re}=\dfrac{\rho aV_o}{\mu_o},\ \mathrm{Pr}=\dfrac{C_p\mu_o}{\kappa},\ F^{\Delta}=\beta^*a,\ \Lambda_1=\dfrac{a^2A^*}{\kappa},\ \Lambda_2=\dfrac{a^2B^*}{\kappa}\end{array}\right\}. \tag{9}$$

Dimensionless Boundary Conditions

$$\left.\begin{array}{ll}y=-1,\ f=\beta_1(e^{-\alpha\theta}f'+2\Omega(f')^3), & \theta=0\\ y=1,\ f=-\beta_2(e^{-\alpha\theta}f'+2\Omega(f')^3), & \theta=0\end{array}\right\}. \tag{10}$$

where Ω is the third-grade fluid material parameter, G is the pressure gradient parameter, α is the variable viscosity parameter, ε is the activation energy parameter, Gr is the Grashof number, β_1 and β_2, are the lower and upper wall slip parameters respectively, δ is the porous medium shape parameter, λ is the Frank-Kamenetsikii parameter, θ is the non-dimensional fluid temperature, x and y are the non-dimensional axial and normal coordinates to the vertical channel, F is the dimensionless fluid velocity, M is the magnetic parameter, Nd is the radiation parameter, F^{Δ} is the Darcy Forchheimer parameter.

3. Numerical Simulation of the Dimensionless Model

To obtain an approximate solution to the dimensionless equations (6) and (8) subject to the boundary condition in equation (10), a 6^{th} order two point Taylor polynomial is utilized as trial function for both velocity (F) and temperature (θ) as used by Oderinu *et al.* [30],

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(ISSN 2992-4421) *EGBEYEMI, H. B *

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$$\begin{aligned}F(y) &= \Gamma_1(y-\pi_1)^6 + \Gamma_2(y-\pi_1)^6(\pi_2-y) + \Gamma_3(y-\pi_1)^6(\pi_2-y)^2 + \Gamma_4(y-\pi_1)^6(\pi_2-y)^3 \\ &+\Gamma_5(y-\pi_1)^6(\pi_2-y)^4 + \Gamma_6(y-\pi_1)^6(\pi_2-y)^5 + \Gamma_7(y-\pi_1)(\pi_2-y)^6 + \Gamma_8(y-\pi_1)^2(\pi_2-y)^6 . \\ &+\Gamma_9(y-\pi_1)^3(\pi_2-y)^6 + \Gamma_{10}(y-\pi_1)^4(\pi_2-y)^6 + \Gamma_{11}(y-\pi_1)^5(\pi_2-y)^6 + \Gamma_{12}(\pi_2-y)^6\end{aligned} \tag{11}$$

and

$$\begin{aligned}\theta(y) &= \zeta_1(y-\pi_1)^6 + \zeta_2(y-\pi_1)^6(\pi_2-y) + \zeta_3(y-\pi_1)^6(\pi_2-y)^2 + \zeta_4(y-\pi_1)^6(\pi_2-y)^3 \\ &+\zeta_5(y-\pi_1)^6(\pi_2-y)^4 + \zeta_6(y-\pi_1)^6(\pi_2-y)^5 + \zeta_7(y-\pi_1)(\pi_2-y)^6 + \zeta_8(y-\pi_1)^2(\pi_2-y)^6 . \\ &+\zeta_9(y-\pi_1)^3(\pi_2-y)^6 + \zeta_{10}(y-\pi_1)^4(\pi_2-y)^6 + \zeta_{11}(y-\pi_1)^5(\pi_2-y)^6 + \zeta_{12}(\pi_2-y)^6\end{aligned} \tag{12}$$

In this instance, $(\pi_1, \pi_2) = (-1, 1)$ is the domain of the problem. Equations (11) and (12) are imposed on the boundary conditions (10) as,

$$F(-1) - \beta_1\left(e^{-\alpha\theta(-1)}F'(-1) + 2\Omega\left(F'(-1)\right)^3\right) = 0. \tag{13}$$

$$F(1) + \beta_2\left(e^{-\alpha\theta(1)}F'(1) + 2\Omega\left(F'(1)\right)^3\right) = 0. \tag{14}$$

$$\theta(-1) = 0. \tag{15}$$

$$\theta(1) = 0. \tag{16}$$

which results to 4 sets of algebraic equations in Γ_i and ζ_i. Next is to obtain the initial approximation $F_o(y)$ and $\theta_o(y)$ which is done by linearizing equations (6) and (8) as

$$F'' - \alpha\theta - \alpha F' - \alpha^2\theta - \operatorname{Re} F' + 6\Omega F' - \delta(F - \alpha\theta) - MF - F^{\Delta}F + Gr\theta\sin(\omega) + G = 0. \tag{17}$$

and

$$\begin{aligned}&\theta'' - \operatorname{Re}\Pr\theta' + \lambda\left(\left(1+\theta+\varepsilon\theta\right) + \tau\left(\left(F' - \alpha\theta + 2\Omega F'\right) + \delta(F - \alpha\theta) + M\theta + F^{\Delta}F\right)\right). \\ &+\left[\Lambda_1\theta + \Lambda_2 F'\right] + Nd\left[\theta'' + \varepsilon\theta + 3\varepsilon(\theta' + \varepsilon\theta)\right] = 0\end{aligned} \tag{18}$$

respectively. The residuals F_R and θ_R associated with velocity F and temperature θ are obtained by substituting the assumed $F(y)$ and $\theta(y)$ into the linearized equations (17) and (18) giving F_R and θ_R. where F_R and θ_R are then minimized using Legendre collocation points ± 0.9815606342, ± 0.9041172564, ± 0.7699026742, ± 0.5873179543, ± 0.3678314990, ± 0.1252334085 [31], to give another sets of algebraic equations which are solved simultaneously with equations (13) – (16) to obtain Γ_i and ζ_i. Substituting the Γ_i and ζ_i into the

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assumed equations (11) and (12) gives $F_o(y)$ and $\theta_o(y)$ as the initial approximations of $F(y)$ and $\theta(y)$ respectively. The process of collocation is repeated iteratively using the sequence for $\zeta = 1, 2, \ldots,$

$$\begin{aligned} &e^{-\alpha\theta_{\eta-1}} F_\eta{}'' - \alpha e^{-\alpha\theta_{\eta-1}} \theta_\eta{}' F_\eta{}' - \operatorname{Re} F_\eta{}' + 6\Omega F_\eta{}'' \left(F_{\eta-1}{}'\right)^2 - \delta e^{-\alpha\theta_{\eta-1}} F_\eta - M\theta_{\eta-1}{}^n F_\eta - F^\Delta F_{\eta-1}{}^3 \\ &+ Gr\theta_\eta \sin(\omega) + G = 0 \end{aligned} \quad (19)$$

and

$$\begin{aligned} &\theta_h{}'' - \operatorname{Re}\Pr\theta_\eta{}' + \lambda\left[\left(1+\varepsilon\theta_{\eta-1}\right)^m e^{\frac{\theta_{\eta-1}}{1+\varepsilon\theta_{\eta-1}}} + \tau\left\{\begin{aligned}&\left(F_{\eta-1}{}'\right)^2\left(e^{-\alpha\theta_{\eta-1}} + 2\Omega\left(F_{\eta-1}{}'\right)^2\right) \\ &+\delta e^{-\alpha\theta_{\eta-1}} F_{\eta-1}{}^2 + F^\Delta F_{\eta-1}{}^3 + M\theta_{\eta-1}{}^n F_{\eta-1}{}^2\end{aligned}\right\}\right] \\ &+ + \left[\Lambda_1\theta_\eta + \Lambda_2 F_\eta{}'\right] + Nd\left[(1+\varepsilon\theta_{\eta-1})^3\theta_\eta{}'' + 3\varepsilon(1+\varepsilon\theta_{\eta-1})^2\theta_{\eta-1}{}'^2\right] = 0 \end{aligned} \quad (20)$$

this iteration continue until

$$\left|\frac{F_\eta - F_{\eta-1}}{F_\eta}\right| = q_1 . \quad (21)$$

and

$$\left|\frac{\theta_\eta - \theta_{\eta-1}}{\theta_\eta}\right| = q_2 . \quad (22)$$

q_1 and q_2 are the tolerance values which should be declining with increase in iteration to justify the convergence of the method employed. On implementing TTPILT on the dimensionless equations (6), (8) and (10) for the parametric values $b = 1.0,\ n = 0, m = 0.5, \lambda = 0.1, \alpha = 0.1, \Omega = 0.1, \delta = 0.1, Gr = 0.1, \omega = 0.1, G = 1.0, \beta_1 = 0.1,$

$\beta_2 = 0.1, M = 0, Nd = 0, \gamma = 0.1, \varepsilon = 0.1, \Lambda_1 = 0, \Lambda_2 = 0, F^\Delta = 0$ the solution for the velocity and temperature profile are given respectively as,

$$\begin{aligned} F(y) &= 1.0 + 54.63735725(y+1)^6(1-y) + 2.562437187(y+1)^6(1-y)^2 - 0.1701538862 \\ &(y+1)^6(1-y)^3 + 0.0006167182032(y+1)^6(1-y)^4 - 2.762548657(y+1)^2(1-y)^6 - 0.1711 \\ &410210(y+1)^3(1-y)^6 - 0.0006167184012(y+1)^4(1-y)^6 - 431.3554098(y+1)^2(1-y)^5 \\ &+ 11.06284506(y+1)^3(1-y)^5 + 0.2567121816(y+1)^4(1-y)^5 + 1600.545850(y+1)^5 \\ &(1-y) - 473.2364434(y+1)^5(1-y)^2 - 10.24690141(y+1)^5(1-y)^3 + 0.2552301792 \\ &(y+1)^5(1-y)^4 + \ldots + 2863.974368(1-y)^3 \end{aligned} . \quad (23)$$

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and

$$\begin{aligned}
\theta(y) &= 5.635399814(y+1)^6(1-y) + 0.4326892584(y+1)^6(1-y)^2 + 0.007915487022 \\
&(y+1)^6(1-y)^3 - 0.4231913305(y+1)^2(1-y)^6 + 0.007915491221(y+1)^3(1-y)^6 - 47.37 \\
&519002(y+1)^2(1-y)^5 + 1.692765291(y+1)^3(1-y)^5 - 0.01187324313(y+1)^4(1-y)^5 - \\
&1075.765154(y+1)^5(1-y) - 50.65335163(y+1)^5(1-y)^2 - 1.730757064(y+1)^5(1-y)^3 \\
&-0.01187322424(y+1)^5(1-y)^4 + 70648.79390(y+1)(1-y)^4 + 992.0087718(y+1)(1-y)^5 \\
&+5.243788374(y+1)(1-y)^6 + 78815.83974(y+1)^4(1-y) + 0.023744896(y+1)^4(1-y)^4 \\
&+57.7927419(y+1)^4(1-y)^3 + 3351.737702(y+1)^4(1-y)^2 + \ldots + 1.121022822\times 10^5(1-y)^3
\end{aligned} \tag{24}$$

4. Discussion of Results

This study has modelled a phenomena that mirrors the flow of a radiative magnetohydrodynamic combustible third-grade fluid characterized by a one step exothermic reaction through a porous filled vertical channel under the combined influence of Darcy Forchheimer inertia, nonlinear thermal radiation and nonuniform internal heat generation through the process of ordinary differential equation and transformed to its dimensionless form via the procedure of similarity transformation technique. The dimensionless equations obtained is solved numerically via the adoption of TTPILT. This section therefore present and discuss the outcomes of the solutions of the dimensionless model in equations (6) and (8) subject to the boundary condition in equation (10). Tables are also adopted to compare the results obtained from TTPILT with that of Spectral Chebyshev Collocation Method (SCCM) [26]. The effect of various flow parameters on velocity and temperature profiles are graphically investigated and physically discussed.

4.1 Convergence of the method of solution

Tables 1 and 2 reveals the relative tolerance for the first four iterations for both velocity and temperature profiles respectively. For the velocity profile, the minimum relative tolerance for the first four iterations are given as 0.9007046210, 0.00776218835 and 0.0057679749950 respectively, while their corresponding maximum relative tolerance values are 0.1172377947, 0.05038625737 and 0.0002416195155 respectively. Also, for the temperature profile, the maximum relative tolerance values are 0.1187562132, 0.095343233777 and 0.001017856434 While the minimum relative tolerance values are 0.0132333675, 0.000900545632 and 0.000222870399 respectively. As a result of the declining of the relative tolerance with increase in the iterations, this implies that the method employed is converging and accurate. The results obtained at the last iteration $F_3(y)$ and $\theta_3(y)$ are compared with Spectral Chebyshev Collocation Method (SCCM) as seen in tables 3 and 4. The absolute errors of both methods are determined. The maximum and minimum absolute errors for velocity profile are 0.0000000210 and 0.0000000002 respectively while that of temperature profile are 0.05315385369 and $1.1006285586\times 10^{-19}$ respectively.

4.2 Discussion of the influence of flow parameters on velocity profile

Raising the Darcy Forchheimer parameter $F^{\Delta} = 0.1, 0.6, 1.1, 1.6$ caused a decline in the fluids motion as shown in figure 2. An increase in the Darcy–Forchheimer parameter, which represents enhanced inertial effects in the porous

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medium, led to a reduction in the velocity of the combustible fluid flowing through the vertically oriented Darcy porous channel. As this parameter rose, the resistance to flow became stronger beyond simple viscous (Darcy) drag, causing a more pronounced deceleration of the fluid. This interplay is critical in engineering systems such as high-temperature packed-bed reactors used in combustion or gasification processes, where reactive fluids pass through

porous catalytic media and experience significant radiative heat transfer and spatially varying heat generation due to exothermic reactions.

A rise in the magnetic parameter $M = 0.5, 1.0, 1.5, 2.0$ introduced a stronger Lorentz force which opposed the motion of the electrically conducting combustible fluid, thereby reducing its velocity throughout the vertically positioned Darcy porous channel as shown in figure 3. This magnetic damping effect thickened the momentum boundary layer and suppressed flow acceleration. While elevated temperatures from radiative effects and internal heat sources decreased the fluid viscosity and promoted motion, the magnetic field's resistive influence dominated, leading to an overall deceleration and more stabilized flow. This behavior is particularly relevant in magnetohydrodynamic (MHD) combustion systems or plasma-assisted burners, where ionized combustible gases flow through porous structures under applied magnetic fields to control flame stability, heat transfer, and reaction rates in advanced energy conversion devices. Figure 4 revealed that increasing the porous medium shape parameter $\delta = 0.1, 0.3, 0.5, 0.7$ reduced the motion of the combustible fluid. Raising the porous medium shape parameter intensified the geometric resistance offered by the porous matrix, which enhanced the effective drag force thereby making the axial velocity profile to diminish near the channel walls where shear and non-Newtonian effects were pronounced. From an engineering view, this phenomena is significant in applications such as enhanced oil recovery in inclined reservoirs, catalytic packed- bed

Table 1: Relative tolerance of the solution obtained for the velocity profile

y	$\left\lvert \dfrac{F_1 - F_o}{F_o} \right\rvert$	$\left\lvert \dfrac{F_2 - F_1}{F_1} \right\rvert$	$\left\lvert \dfrac{F_3 - F_2}{F_2} \right\rvert$
-1.00	0.1172377947	0.05038625737	0.0002416195155
-0.75	0.0607851587	0.02086404750	0.0003576982591
-0.50	0.0143481012	0.01341181908	0.0007507822827
-0.25	0.0960508251	0.00522429291	0.0007361634703
0.00	0.1921165564	0.00362961121	0.0001109673432
0.25	0.3286095058	0.01138336841	0.0000762644913
0.50	0.6035354636	0.01637769107	0.0000353165986
0.75	0.8952048690	0.01613372519	0.0004441210140
1.00	0.9007046210	0.00776218835	0.0057679749950

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"Thermal Analysis of the Effect of Non-Linear Thermal Radiation and Non-Uniform Internal Heat Generation on a MHD Combustible Third-Grade Fluid Flowing Through a Vertically Positioned Darcy Porous Channel"

Table 2: Relative tolerance of the solution obtained for the temperature profile

y	$\left\lvert\dfrac{\theta_1-\theta_o}{\theta_o}\right\rvert$	$\left\lvert\dfrac{\theta_2-\theta_1}{\theta_1}\right\rvert$	$\left\lvert\dfrac{\theta_3-\theta_2}{\theta_2}\right\rvert$
-1.00	0.0132333675	0.000900545632	0.000222870399
-0.75	0.0232547140	0.000452323952	0.000219854924
-0.50	0.0078413711	0.002027560101	0.000461167006
-0.25	0.0094399514	0.000820510742	0.000557174898
0.00	0.0282436410	0.001096912223	0.001014194350
0.25	0.0484380333	0.003792861380	0.002254896812
0.50	0.0700215593	0.008346139462	0.005558754367
0.75	0.0931117259	0.002955582419	0.000967180479
1.00	0.1187562132	0.095343233777	0.001017856434

Table 3: Comparison of the results obtained from the present work with Spectral Chebyshev Collocation Method (SCCM) for Temperature $F(y)$ where $b=1.0,\ n=0, m=0.5, \lambda=0.1, \alpha=0.1,$

$\Omega=0.1, \delta=0.1, Gr=0.1, G=1.0, \beta_1=0.1, \beta_2=0.1, M=0,\ Nd=0, \Gamma=0, \varepsilon=0.1, \gamma=0.1,$

$F^{\Delta}=0, \Lambda_1=0, \Lambda_2=0$

y	TTPILL	SCCM [11]	Absolute difference of TTPILL and SCCM
-1.00	0.0934004271796298	0.093400427310130	0.00000000013
-0.75	0.278270247555746	0.278270251882776	0.0000000043
-0.50	0.418108755462444	0.418108756861981	0.0000000014
-0.25	0.507363722172939	0.507363719780844	0.0000000024
0.00	0.540803241406083	0.54080322117084	0.0000000202
0.25	0.515175577361859	0.515175556370712	0.0000000210
0.50	0.430592512405898	0.4305925030480927	0.0000000094
0.75	0.290250676231661	0.29025067501405133	0.0000000012
1.00	0.0988679977072932	0.09886799791001163	0.0000000002

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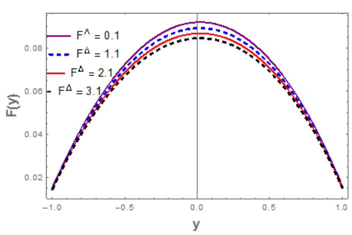
Table 4: Comparison of the results obtained from the present work with Spectral Chebyshev Collocation Method (SCCM) for Temperature $\theta(y)$ where $b=1.0,\ n=0, m=0.5, \lambda=0.1, \alpha=0.1,$

$\Omega=0.1, \delta=0.1, Gr=0.1, G=1.0, \beta_1=0.1, \beta_2=0.1, M=0,\ Nd=0, \Gamma=0, \varepsilon=0.1, \gamma=0.1,$

$F^{\Delta}=0, \Lambda_1=0, \Lambda_2=0$

y	TTPILL	SCCM [11]	Absolute difference of TTPILL and SCCM
-1.00	0.0000000000000000	$1.1006285586\times10^{-19}$	$1.1006285586\times10^{-19}$
-0.75	0.0227208433102568	0.02272208431322944	0.00000124100
-0.50	0.0392380869745187	0.039238086997582716	0.00000000003
-0.25	0.0494275244357657	0.049427524412263134	0.00000000003
0.00	0.0531538537201620	0.053153853688921044	0.05315385369
0.25	0.0502640853345581	0.050264085271171340	0.05026408527
0.50	0.0405813346491640	0.040581334644545750	0.04058133464
0.75	0.0239025595292789	0.023902559552278567	0.00000000002
1.00	0.0000000000000000	$1.4543862558\times10^{-18}$	$1.4543862558\times10^{-18}$



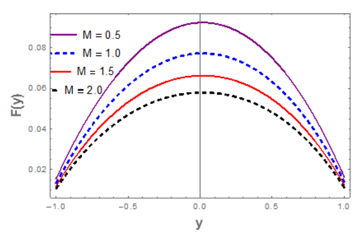


Figure 2: Varying Darcy Forchheimer parameter on velocity profile

Figure 3: Varying magnetic parameter on velocity profile

reactors, geothermal energy extraction systems, and MHD power generators, where controlling porous structure geometry directly affects flow rate, heat transfer efficiency, and reaction intensity. Optimizing the porous medium shape parameter can therefore be used to regulate residence time, improve thermal management, suppress runaway exothermic reactions, and enhance overall system stability and safety in industrial transport processes involving complex non-Newtonian conducting fluids.

Figure 5 shows that a rise in the pressure gradient parameter $G=0.2, 0.3, 0.4, 0.5$ strengthened the driving force acting on the combustible fluid, flowing through the vertically oriented Darcy porous channel thereby leading to a higher axial velocity as the fluid is pushed more forcefully through the porous matrix, overcoming drag imposed by the medium. In the presence of nonlinear thermal radiation and nonuniform internal heat generation, the effect became more complex but typically reinforcing. Increased pressure-driven flow enhanced convective transport, while elevated temperatures from internal heat sources reduced fluid viscosity and amplified buoyancy forces, further accelerating the flow. This behavior is highly relevant in systems such as vertical packed-bed

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(ISSN 2992-4421) *EGBEYEMI, H. B *

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combustion reactors used in chemical processing or energy generation, where pressurized combustible gases flow through heated porous catalysts, and careful control of pressure gradients is essential to regulate flow rates, ensure efficient heat removal, and avoid dangerous hotspots or combustion instability. A rise in the third-grade material parameter $\Omega = 0.1, 3.1, 6.1, 9.1$ increased resistance to deformation, which ted to suppression in the velocity of the combustible fluid as it flows through the vertical Darcy porous channel as demonstrated in figure 6. As this parameter grew, higher-order stress contributions became more significant, effectively thickening the fluid and damping its motion despite the driving forces. Under nonlinear thermal radiation and nonuniform internal heat generation, temperature elevations can partially offset this resistance by lowering the apparent viscosity and enhancing buoyancy, but the dominance of the strong third-grade parameter still reduced the velocity profile, thereby making it more uniform across the channel. This phenomenon is relevant in polymer-laden or reactive fuel flows in porous catalytic reactors or thermal insulation systems in propulsion devices, where non-Newtonian combustible mixtures pass through heated porous structures and accurate prediction of velocity is crucial for controlling heat transfer, reaction rates, and system stability. Elevating the Grashof number $Gr = 0.1, 2.1, 4.1, 6.1$ enhanced the velocity of the combustible fluid in the vertical Darcy porous channel because stronger thermal buoyancy drives more vigorous upward motion as revealed in figure 7. The presence of nonlinear thermal radiation and nonuniform internal heat generation makes this effet more pronounced, because radiation intensified heat transfer at elevated temperatures, steepened thermal gradients, while internal heat sources elevated local temperatures and further amplified density differences that powered buoyancy which made the flow accelerated. Overall, a higher Grashof number shifted the flow toward buoyancy-dominated convection, increasing velocity and potentially altering stability. This scenario is crucial in vertical packed-bed combustion systems or geothermal energy extraction columns, where heat-induced natural convection through porous media governs fluid transport, and managing buoyancy-driven flow is critical to avoid overheating, ensure efficient heat removal, and maintain safe operation. An increase in the temperature-dependent heat generation parameter $\Lambda 1 = 0.0, 0.5, 1.0, 1.5$ intensified the internal thermal energy produced within the combustible fluid, which raised the fluid temperature and consequently enhanced buoyancy forces in the vertical Darcy porous channel as revealed in figure 8. Considering the combined effects of nonlinear thermal radiation and nonuniform internal heat generation, this elevated thermal state reduced the effective fluid viscosity and accelerated the flow velocity. The nonlinear radiation mechanism further amplified heat transfer at higher temperatures, causing a stronger coupling between momentum and energy transport, while the porous Darcy medium moderated the motion through drag resistance. As a result, the velocity profile increased with higher temperature-dependent heat generation, although excessive heating may lead to thermal instability or localized combustion enhancement within the channel. This phenomenon is highly relevant in engineering systems such as catalytic reactors and porous combustion chambers used in gas turbines or industrial furnaces, where combustible gases flow through porous structures and internal heat generation significantly affects flame stability, heat transfer efficiency, and fluid transport characteristics.

An increase in the space-dependent heat generation parameter $\Lambda 2 = 0.0, 0.5, 1.0, 1.5$ heightened the internal thermal energy produced within the vertically stationed Darcy porous channel, causing the temperature of the combustible fluid to rise more rapidly along the flow direction as shown in figure 9. Under the combined effects of nonlinear thermal radiation and buoyancy forces, this elevated temperature reduces fluid density and strengthened the thermal buoyancy force, which in turn accelerated the fluid velocity near the heated regions of the porous medium.

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(ISSN 2992-4421)

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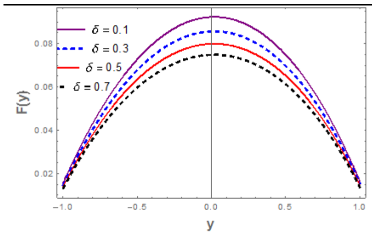


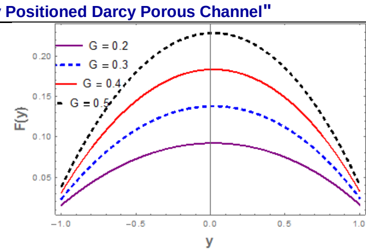
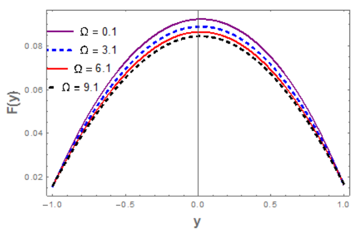


Figure 4: Varying porous medium shape parameter on velocity profile

Figure 5: Varying pressure gradient parameter on velocity profile



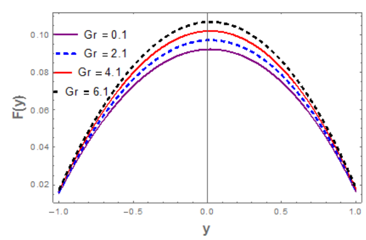
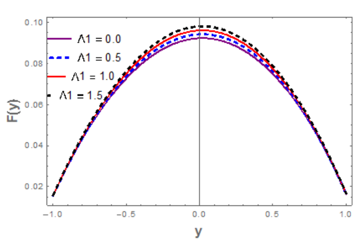


Figure 6: Varying third-grade material parameter on velocity profile

Figure 7: Varying Grashof number on velocity profile



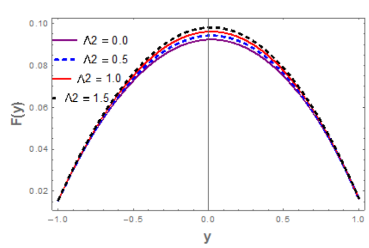


Figure 8: Varying temperature dependent heat generation parameter on velocity profile

Figure 9: Varying space dependent heat generation parameter on velocity profile

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https://ijojournals.com/ Volume 09 || Issue 07 || July, 2026 ||

Because nonlinear radiation contributed additional radiative heat transport at high temperatures, the thermal boundary layer became thicker and the momentum boundary layer is enhanced, further increasing the flow velocity. However, if the heat generation becomes excessively large, the combustible fluid may experience strong thermal gradients that can destabilize the flow and increase the likelihood of ignition or thermal runaway within the porous structure. This phenomenon occurs in engineering systems such as catalytic reactors and packed-bed combustion chambers used in energy conversion and chemical processing industries, where combustible gases flow through porous matrices subjected to intense internal heat generation and radiative heat transfer. Figure 10 demonstrated that an enhancement in the Frank-Kamenetskii parameter $\lambda = 0.1, 0.3, 0.5, 0.7$ signified a stronger exothermic chemical reaction within the combustible fluid, leading to greater heat release inside the Darcy porous channel. As the fluid temperature rose, nonlinear thermal radiation enhanced radiative heat transfer, while the nonuniform internal heat generation further intensified the thermal field throughout the porous medium. The resulting temperature increase strengthened buoyancy forces due to density variations, thereby accelerating the fluid velocity along the vertical channel. Consequently, both the thermal and momentum boundary layers became thicker, and the flow rate increased significantly as the Frank-Kamenetskii parameter grew. However, the rapid heat accumulation may trigger thermal instability or combustion runaway, especially in highly reactive fluids confined within porous structures. This behavior is particularly important in engineering applications such as underground coal gasification systems, catalytic packed-bed reactors, and porous burner technologies, where reactive combustible fluids flow through porous materials under strong internal heating and radiative effects.

4.3 Discussion of the influence of flow parameters on temperature profile

Figure 11 demonstrated that raising the Frank-Kamenetskii parameter $\lambda = 0.1, 0.3, 0.5, 0.7$ intensified the exothermic chemical reaction within the combustible fluid, leading to a higher rate of internal heat production in the Darcy porous channel. As this parameter grew, the fluid temperature increased significantly because the generated heat exceeded the rate at which heat can be dissipated through conduction, convection, and porous-medium diffusion. In the presence of nonlinear thermal radiation, the temperature rise became more pronounced since radiative heat transfer strongly depends on temperature and amplifies thermal energy transport at elevated temperatures. Furthermore, nonuniform internal heat generation caused localized hot spots within the porous structure, producing steeper temperature gradients and potentially triggering thermal runaway or ignition in highly reactive regions. The combined effects enhanced buoyancy-driven flow and altered the thermal boundary layer characteristics throughout the channel. This phenomenon is relevant in engineering applications such as catalytic packed-bed nuclear waste repositories and combustion chambers, where reactive fluids flow through porous insulation or catalyst matrices where careful thermal management is required to prevent overheating, material degradation, or spontaneous combustion. Figure 12 revealed that an increase in the temperature-dependent heat generation parameter $\Lambda 1 = 0.0, 0.5, 1.0, 1.5$ caused a substantial rise in the temperature of the combustible fluid flowing through the Darcy porous channel because the internal heat produced by the chemical reaction became stronger as the fluid temperature increased. This created a positive feedback mechanism in which higher temperatures accelerated heat generation, thereby intensifying the thermal field within the porous medium. Under nonlinear thermal radiation, the effect became more significant because radiative heat transfer varied nonlinearly with temperature and contributed additional thermal energy at elevated temperatures. The presence of nonuniform internal heat generation further enhanced localized heating, leading to the formation of thermal hot spots and steeper temperature gradients across the channel. Consequently, the fluid experienced stronger buoyancy effects, thicker thermal boundary layers, and a greater possibility of thermal instability or runaway combustion if cooling is insufficient. This behavior is important in engineering systems such as Packed-bed reactor used in catalytic combustion and petrochemical processing, where reactive fluids move through porous catalyst beds and temperature-sensitive heat generation must be carefully controlled to avoid reactor overheating, catalyst damage, or accidental ignition.

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(ISSN 2992-4421) *EGBEYEMI, H. B *

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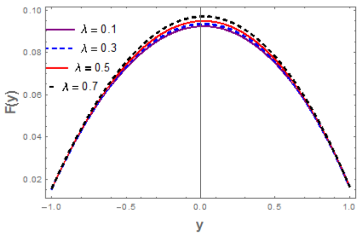
Elevating the space-dependent heat generation parameter $\Lambda2 = 0.0, 0.5, 1.0, 1.5$ intensified the internal thermal energy produced within the porous channel, thereby raising the temperature distribution of the combustible fluid as it flows upward through the vertically positioned Darcy porous medium as shown in figure 13. Because the heat source varied spatially, regions of the channel with stronger heat generation experienced greater thermal buildup, which enhanced buoyancy effects and accelerated thermal diffusion into the fluid. In the presence of nonlinear thermal



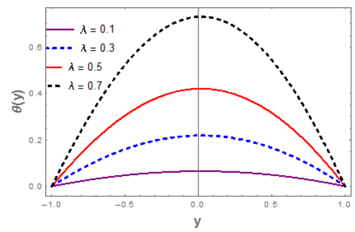
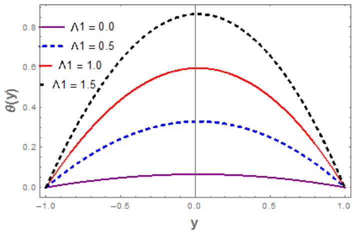


Figure 10: Varying Frank-Kamenetskii parameter on velocity profile

Figure 11: Varying Frank-Kamenetskii parameter on temperature profile



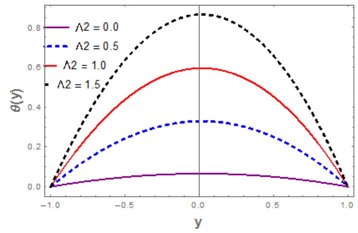


Figure 12: Varying temperature dependent heat generation parameter on temperature profile

Figure 13: Varying space dependent heat generation parameter on temperature profile

radiation, this temperature rise became more pronounced since radiative heat transfer grew rapidly at higher temperatures, leading to thicker thermal boundary layers and elevated fluid temperatures throughout the channel. Also for the combustible fluid, the combined action of nonuniform internal heating and radiation significantly increased the likelihood of thermal runaway or ignition if heat removal is insufficient. This phenomenon is relevant in engineering systems like catalytic reactors packed with porous materials, where exothermic chemical reactions generate nonuniform heat within the reactor bed while thermal radiation and natural convection influence the transport of heat and reactive combustible gases. An enhancement in the activation energy parameter

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(ISSN 2992-4421) *EGBEYEMI, H. B *

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$\varepsilon = 0.1, 0.3, 0.5, 0.7$ in the combustible fluid reduced the reaction rate, which weakened the exothermic heat release driving the temperature field within the vertically oriented Darcy porous channel as demonstrated in figure 14. As a result, the fluid temperature decreased and the thermal boundary layer became less intense, although the exact profile was strongly modulated by nonlinear thermal radiation, which enhanced radiative heat loss at higher temperatures and thereby further suppressed peak thermal levels, and by nonuniform internal heat generation, which locally offset this cooling depending on its spatial distribution. In the vertically aligned porous medium, buoyancy-assisted upward flow interacted with Darcy сопротивance, so changes in heat release directly reshaped convection strength and temperature stratification along the channel. Engineering-wise, this behavior is relevant in packed-bed catalytic combustors or porous burners used in industrial furnaces, where controlling activation energy (via fuel composition or catalyst properties) affects combustion intensity, thermal radiation losses, and internal heat generation distribution, ultimately determining safe operating temperatures and thermal stability of the system. Figure 15 revealed that an increase in radiation parameter $Nd = 0.0, 0.3, 0.6, 0.9$ intensified the contribution of thermal radiation relative to conduction in the energy transport within the combustible fluid flowing through the Darcy porous channel. Also, under nonlinear thermal radiation, this effect became even more pronounced at higher temperature due to the strong temperature dependence of radiative heat flux. As the radiation parameter rose, radiative energy exchange was enhanced, which strengthened heat removal from hotter regions and redistributed thermal energy more effectively throughout the porous matrix, leading to a reduction in peak temperature and a more uniform temperature profile, even in the presence of nonuniform internal heat generation that locally injected thermal energy. In the vertically aligned Darcy medium, buoyancy-driven flow interacted with porous resistance, so the enhanced radiative transport also weakened thermal stratification by smoothing temperature gradients along the channel. This behavior is crucial in engineering systems such as packed-bed catalytic combustors or porous radiant burners used in industrial furnaces, where controlling radiative transfer is crucial for maintaining uniform temperature fields, preventing thermal runaway, and improving overall combustion efficiency and material durability.

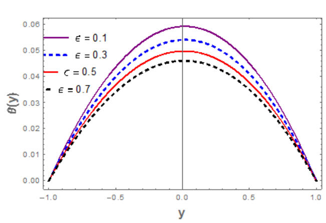

Figure 14: Varying activation energy parameter on temperature profile

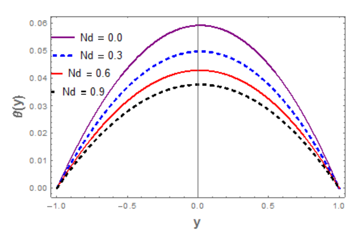

Figure 15: Varying radiation parameter on temperature profile

5. Conclusion and Recommendation

In this study, the thermal behavior of a combustible third-grade fluid flowing through a vertically aligned channel under the combined influence of Darcy–Forchheimer inertia, nonlinear thermal radiation, and non-uniform internal heat generation was investigated. The governing equations representing the physical model were transformed into a system of nonlinear ordinary differential equations through the application of similarity transformation techniques. The resulting equations were solved using a semi-analytical method known as the Two-Point Taylor Polynomial Iterative Linearization Technique with Legendre nodes (TTPILT). The influences of the pertinent flow parameters on the velocity and temperature profiles were thoroughly examined. Based on the findings of this investigation, the following conclusions were established.

1. Raising the Darcy Forchheimer parameter, third-grade material parameter, porous media shape parameter and magnetic parameter caused a decline in the velocity profile while the opposite trend was noticed for the velocity profile when the temperature dependent heat generation parameter, space dependent heat generation parameter, Frank-Kamenetskii parameter and Grashof number were enhanced.
2. Enhancing the temperature dependent heat generation parameter, space dependent heat generation parameter, and Frank-Kamenetskii parameter elevated the temperature profile while a decrease in temperature profile was observed when the activation energy parameter and radiation parameter were raised.

The findings of this study suggest that engineers should carefully monitor the parameters affecting fluid flow, to ensure the safety, stability, and efficient performance of combustion-based systems. Neglecting these factors may lead to excessive heat generation and accelerated reaction rates that can adversely affect system operation. Furthermore, the present theoretical model can be extended to three-dimensional non-Newtonian fluid flow problems by incorporating additional physical parameters for a more realistic and comprehensive analysis.

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