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"Logistic Growth Model of Field Crops with Pest Control Measures"

Logistic Growth Model of Field Crops with Pest Control Measures

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Abstract

In this paper a logistic growth model of field crops is formulated and analysed to examine the best measure to reduce pest infestation to enhance food security. A control model is developed based on three control measures; the prevention measure (u_1) that protect susceptible crops from pest (the rate of brush trimming and removal of weeds), pesticide control measure (u_2) and the rate of removal of infected plants (u_3). The effective reproduction number that determines pest extinction and crop survival is obtained and use to establish the condition for the local asymptotic stability of the pest-free equilibrium using the method of linearization. The existence of the pest- endemic equilibrium is verified using the Descartes rule of signs. Numerical simulations are performed to reveal the effects of the control measures on the field crops. The findings show that each integrated measure is able to mitigate pest infestation in the specified time.

Keywords: *Logistic Growth Model, Field Crops, Control Measures, Effective Reproduction number, Pest*

1. Introduction

Field crops are one of the major sources of food found in most developed countries today. Crops like maize are mostly grown in east African countries, rice, wheat, soybeans and cotton in Asian countries (Gadisa, 2016). They play vital role in global economy and in the environment and have helped in alleviating the effect of food insecurity mostly in African countries. One important factor that affects field crop production is pest infestation. About 30% of annual crop damage is attributed to pest and rodents (Savary et al., 2019). It has been estimated that, pest infestation leads to 40% crop loss yearly and have serious effect on the food security for the ever-increasing population of a country (Wondifraw et al 2021). According to UN Food and Agricultural Organisation, there is danger of food crisis as result of ravaging crops and pastures by locusts and swarms especially in East Africa (FAO, 2020). These pests include insects, weeds nematodes, fungi, viruses, rodents, birds etc.

Since the eighteen centuries, the development of mathematical models has been critical to provide framework, understanding and decision-making processes regarding intervention programs for disease control (Sweileh, 2022; Uwakwe et al., 2020), price options and other dynamical systems. Logistic models have been successfully applied to several population dynamics with great measure of success. The model construction is favourable in unravelling the growth and fall in the number of disease cases around the inflection point. In a study by (Jain et al.,2020), logistic growth model was used to dissect the crucial driving factors that resulted in the rapid disease transmission in India, in order to refine the measures taken to control the pandemic and improve disease. It was also applied tothecovid-19 in China to review the epidemic virus growth and decline. The model summarized the outbreak dynamics using three parameters that characterize the epidemic's timing, rate and peak

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(Zou et al., 2020). The study in (Abusam et al.,2020) used the data for Kuwait to assess the adequacy of the two most commonly used logistic models (Verhulst and Richards models) for describing the dynamics COVID-19. Specifically, the study assessed the predictive performance of these two models and the practical identifiability of their parameters. A time-delayed SIR epidemic model with a logistic growth of susceptible, Crowley–Martin type incidence, and Holling type III treatment rates was proposed and analysed mathematically in a work by (Goel et al 2022).

Ordinary differential equations (ODEs) have been used to model plants infected with viruses. For instance, the authors in (Shi et al., 2014; Hugo et al., 2019) formulated and analysed the dynamics of a vector-borne plant disease model. An eco-epidemiological deterministic model for the transmission dynamics of maize MSD in maize plant was developed and analysed by (Alemneh et al., 2019). The study in Anggriani et al., 2018 developed a mathematical model of plant disease with the effect of fungicide and the optimal control was obtained.

However, logistic growth model of field crops with pest control measures are yet to be developed and analysed. Therefore, in this study a logistic growth model incorporating three control measures is formulated and analysed to ascertain the best control measure to reduce and eliminate pest infestation in order to increase the yield of field crops and promote food security.

2. Model formulation

The disease transmission considers two different populations, the field crop population $N_1(t)$ and the pest population $N_2(t)$. The field crop population has two sub classes: susceptible and infected while the pest population has one class. At time t, S(t) denotes the density of the susceptible field crop, and I(t) denotes the density of the infected field crop, so that $N_1(t) = S(t) + I(t)$. The susceptible pest density is denoted by P(t). In the absence of pest infestation, the field crop population grows logistically with intrinsic growth rate r and environmental carrying capacity $K(K > 0)$. The susceptible pests are recruited at rate Λ. The natural death rate for susceptible pest is μ_2. Susceptible plants move to the infected subclass following contacts pest infestation at a per capita rate β. The field crop dies at the rate μ_1. The susceptible field crops and pest population have the Holling type functional response terms $\frac{\beta SP}{1+\alpha P}$, and $\frac{qI}{1+wI}$ respectively, which is saturated with the susceptible and infected field crops, where α and ware the saturation factors that measures the inhibitory effect. The model equations are given as;

$$\begin{aligned}
\dot{S} &= rs\left(1-\frac{S}{K}\right)-\frac{\beta SP}{1+\alpha P} \\
\dot{I} &= \frac{\beta SP}{1+\alpha P}-\frac{qI}{1+\alpha I}-\mu_1 I \qquad (1.1) \\
\dot{P} &= \Lambda+\frac{qI}{1+\alpha I}-\mu_2 P
\end{aligned}$$

with nonnegative initial conditions $S(0) = S_0$, $I(0) = I_0$, $P(0) = P_0$.

Introducing time dependent controls on the model (1), to identify policies that control pest. We apply control measures on the model with the following assumptions. The first measure is prevention measure (u_1) that protect susceptible crops from pest. Hence, it minimizes or eliminate pest-susceptible crops contacts by a factor (1- u_1). The control function u_2 represents pesticide control. The pesticide harms the entire pest population raising their mortality rate. The third control variable u_3 is the removal of infected crops (uprooting). After incorporating the controls u_1, u_2 and u_3 in the model (1.2), we obtain the control model,

$$\begin{aligned}
\dot{S} &= rs\left(1-\frac{S}{K}\right)-(1-u_1)(1-u_2)\frac{\beta SP}{1+\alpha P} \\
\dot{I} &= (1-u_1)(1-u_2)-\frac{\beta SP}{1+\alpha P}-(1-u_3)\frac{qI}{1+\alpha I}-\mu_1 I \\
\dot{P} &= \Lambda+(1-u_3)\frac{qI}{1+\alpha I}-(u_3+\mu_2)P
\end{aligned} \tag{1.2}$$

with nonnegative initial conditions $S(0) = S_0$, $I(0) = I_0$, $P(0) = P_0$.
The description of the parameters is found in Table 1.

Table 1: Description of Parameters

Parameters	Description
r	Intrinsic growth rate of field crops
$\boldsymbol{\Lambda}$	Recruitment rate of pests
$\boldsymbol{\beta}$	Predation rate of pest on field crops
q	maximum rate of pest infestation
$\boldsymbol{\mu_1}$	Death rate of infected field crops
$\boldsymbol{\mu_2}$	Death rate of pests
$\boldsymbol{\alpha}$, w	Saturation factors

3. The Equilibrium States

3.1 Pest- free equilibrium (PFE) and Effective reproduction number

Examining the dynamics of the model (1.2) by the available fixed points. The pest-free equilibrium denoted by $E_0$is obtained as:

$$E_0 = (S^0, I^0, P^0) = (K,0,0)$$

In order to find the stability analysis of the model (1.2), the effective reproduction number (R_0) of the model (1.2) is obtained using the next generation method (Pauline & Watmough, 2002).

$$R_0 = \frac{(1-u_1)(1-u_2)\beta Kq}{(u_2+\mu_2)(q+\mu_1)}$$

Theorem 3.1.: If R0 < 1, the disease-free equilibrium P0 of the system (1.2) is locally asymptotically stable. This result is based on a theorem in Van den Driessche and Watmough (2002). The Jacobian matrix for (1.2) is given as

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$$J=\begin{pmatrix} -r-\frac{2rs}{K}-\frac{(1-u_1)(1-u_2)\beta P}{1+\alpha P} & 0 & \frac{\alpha(1-u_1)\beta SP-(1+\alpha P)(1-u_1)\beta S}{(1+\alpha P)^2} \\ \frac{(1-u_1)(1-u_2)\beta P}{1+\alpha P} & \frac{WqI-q(1+wI)}{(1+wI)^2}-\mu_1 & \frac{(1+\alpha P)(1-u_1)\beta S-\alpha(1-u_1)\beta SP}{(1+\alpha P)^2} \\ 0 & \frac{q(1+wI)-wqI}{(1+wI)^2} & -(u_3+\mu_2) \end{pmatrix}$$

Evaluating at the Pest- free equilibrium to get the eigen values

$$J_{E0}=\begin{pmatrix} -r & 0 & (1-u_1)\beta K \\ 0 & -(q+u_1) & (1-u_1)\beta K \\ 0 & q & -(u_2+\mu_2) \end{pmatrix}$$

$$(-r-\lambda)[(q+\mu_1+\lambda)(\mu_2+u_3+\lambda)-q(1-u_1)\beta K]=0$$

$$(q+\mu_1)(u_3+\mu_2)+\lambda u_2+\lambda\mu_2+\lambda^2-q(1-u_1)\beta K=0$$

$$\lambda^2+(u_3+\mu_2)\lambda+(q+\mu_1)(u_3+\mu_2)-q(1-\mu_1)\beta K=0$$

$$\lambda^2+(u_3+\mu_2)\lambda+(q+\mu_1)(u_3+\mu_2)-(q+\mu_1)(u_2+\mu_2)R_0=0$$

$$\lambda^2+(u_3+\mu_2)\lambda+(q+\mu_1)(u_3+\mu_2)(1-R_0)=0 \qquad (3.1)$$

From (3.1), by method of linearization, a_0 =1 > 0, a_1 = $(u_2+\mu_2)>0$ while a_2= $(q+\mu_1)(u_3+\mu_2)(1-R_0)$> 0 iff R_0< 1. Hence the pest free equilibrium is local asymptotically stable whenever R_0< 1.

3.2 Existence of Pest- Endemic equilibrium (PEE)

The pest –endemic point for the model (1.2) is given by

$$S^*=\frac{k(r-c_1\lambda^*)}{r}$$

$$I^*=\frac{c_1c_2Kr\lambda^*-Kc_1^2c_2{\lambda^*}^2}{r(q+\mu_1c_2)} \qquad (3.2)$$

$$P^*=\frac{rc_1(q+\mu_1c_1)\Lambda+qc_1c_2Kr\lambda^*-qKc_1^2c_2{\lambda^*}^2}{rc_2(q+\mu_1c_2)(u_2+\mu_2)}$$

But,

$$\lambda^*=\frac{\beta P^*}{1+\alpha P^*} \qquad (3.3)$$

Substituting P^* in (3.2)into (3.3) gives the polynomial;

$$A_0{\lambda^*}^3+A_1{\lambda^*}^2+A_2\lambda^*+A_3=0 \qquad (3.4)$$

Where $A_3=c_1c_2Kr\Lambda(1-R_0)$

Using Descartes' rule of signs to discuss the existence of possible positive roots of (3.4). The results are summarized in Table 2. The following remark summarizes the results in Table 2
The following can be established

Remark 3.1

- If all the coefficients of the polynomial (3.4) have the same signs, then the model system (1.2) has no endemic equilibrium point,
- If $R_0\neq 1$, then the system has zero, one, two or three endemic equilibrium points,
- If R_0= 1, then the system has either zero, one or two endemic equilibrium points.

The last two conditions above imply that the model system (1.2) could exhibit the phenomenon of backward bifurcation, when a stable PFE co-exists with a stable PE equilibrium. This is an epidemiological situation in which the classical requirement of having

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the effective reproduction number less than unity although necessary is not sufficient to eliminate pest infestation.

Table 2: Number of possible positive roots of equation (3.4)

Case	A_3	A_2	A_1	A_0	Possible positive root	R_0 Condition
(1)	+	+	+	-	1	$R_0 < 1$
(2)	+	+	+	+	0	$R_0 > 1$
(3)	+	+	-	-	1	$R_0 < 1$
(4)	+	+	-	+	0 or 2	$R_0 > 1$
(5)	+	-	+	-	1 or 3	$R_0 < 1$
(6)	+	-	+	+	0 or 2	$R_0 > 1$
(7)	+	-	-	-	1	$R_0 < 1$
(8)	+	-	-	+	0 or 2	$R_0 > 1$
(9)	-	+	+	-	0 or 2	$R_0 < 1$
(10)	-	+	+	+	1	$R_0 > 1$
(11)	–	+	-	-	0 or 2	$R_0 < 1$
(12)	–	+	-	+	1 or 3	$R_0 > 1$
(13)	–	-	+	-	0 or 2	$R_0 < 1$
(14)	–	-	+	+	1	$R_0 > 1$
(15)	–	-	--	-	0	$R_0 < 1$
(16)	–	-	-	+	1	$R_0 > 1$

4. Numerical simulations

In this section, we studied numerically the effects of the control measures u_1 (prevention), Pesticide control (u_2) and removal of infected field crops (u_3). We considered the effect of each control measure and the combination of two control measures using S(0) = 500, I(0)=20, P(0)=100 and the parameter values in Table 3.

Table 3: Parameter values

Parameters	Value	Source
r	0.005	Alemneh et al 2000
Λ	0.02	Alemneh et al 2019
β	0.04	Bosque-Perez, 2000
q	0.45	Bosque-Perez, 2000
μ_1	0.008	Alemneh et al 2019
μ_2	0.0303	Alemneh et al 2019
α, w	0.6, 0.4	Alemneh et al 2019

4.1 Effect of the Control measures

The effect of the control measures is investigated numerically, taking each control individually and then incorporating more than one measure.

Strategy A: Prevention measure (u_1)≠0, Pesticide control (u_2) = 0, Removing infected crops (u_3) = 0

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The implementation of prevention measures alone has a positive impact on reducing the number of field crops infested by pest as depicted in figure1.

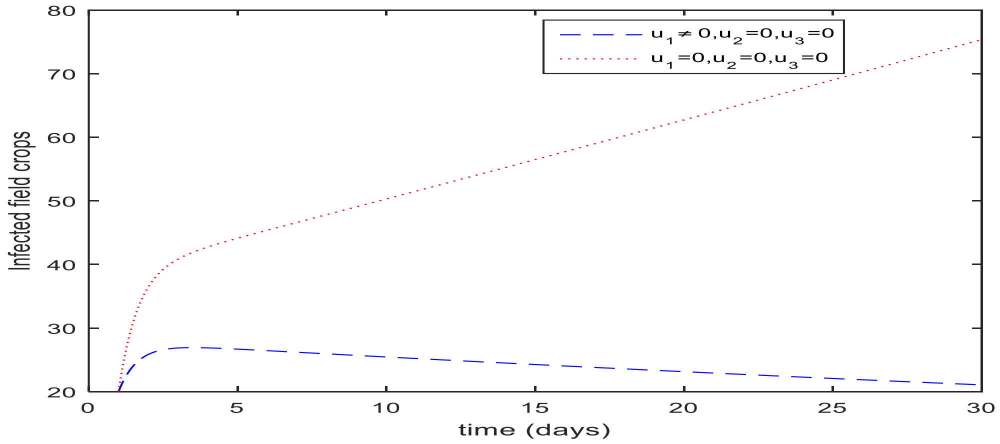

Figure 1: Simulation of infected crops with preventive control measure (u_1)

Strategy B: Pesticide control (u_2)$\neq$ 0,prevention measure (u_1) = 0, removing infected crops (u_3) = 0

The implementation of pesticide as the only control measure is not sufficient enough in reducing the number of crops infested by pest as shown in figure 2.

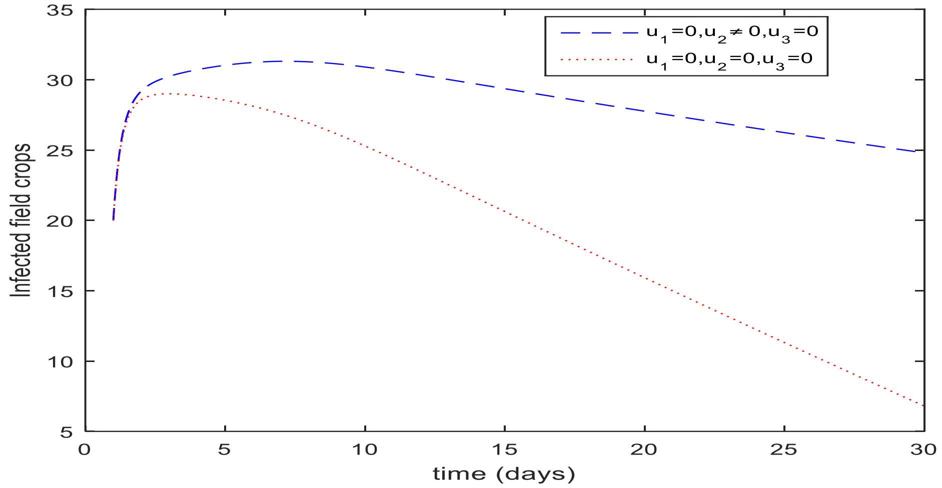

Figure 2: Simulation of infected crops with pesticide control measure (u_2)

Strategy C: Removal of infected crops(u_3)$\neq$0, prevention measure (u_1)=0, removing infected crops(u_3) = 0.

Removing infected crops will reduce the number of other infested crops to 5 in 30days with initial number of infected crops as 20.

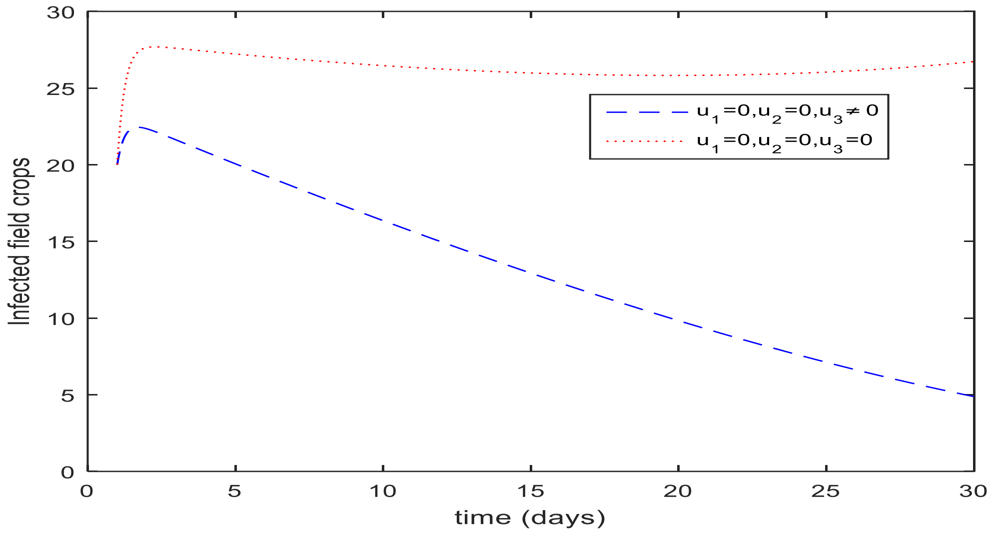

Figure 3: Simulation of infected crops with removal of infected crops (u_3)

Strategy D: Prevention measure (u_1)$\neq 0$, Pesticide control (u_2) $\neq 0$, removal of infected crops (u_3) = 0

From figure 4, the combination of the prevention measure and the use of pesticide will reduce the of crops infected to about 35 starting with 20 infected crops in 30days.

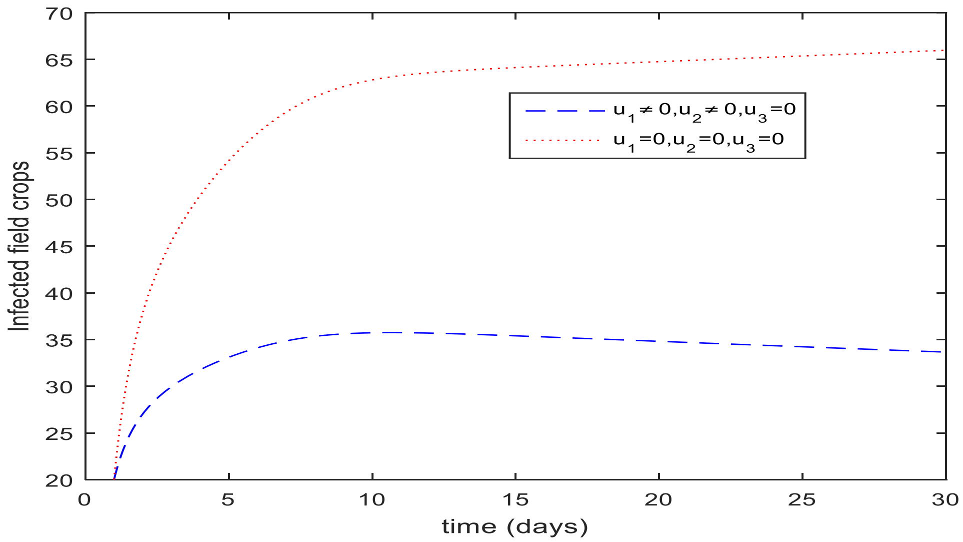

Figure 4: Simulation of infected crops with prevention measure (u_1)$\neq 0$, pesticide control (u_2) $\neq 0$

Strategy E: Prevention measure (u_1)$\neq 0$, Pesticide control (u_2) = 0, removal of infected crops (u_3) $\neq$0

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The implementation of prevention and removing of infected crops will in the long run bring the number of infested crops to about its initial stating point at about 30 days after pest infestation. This can be seen in figure 5.

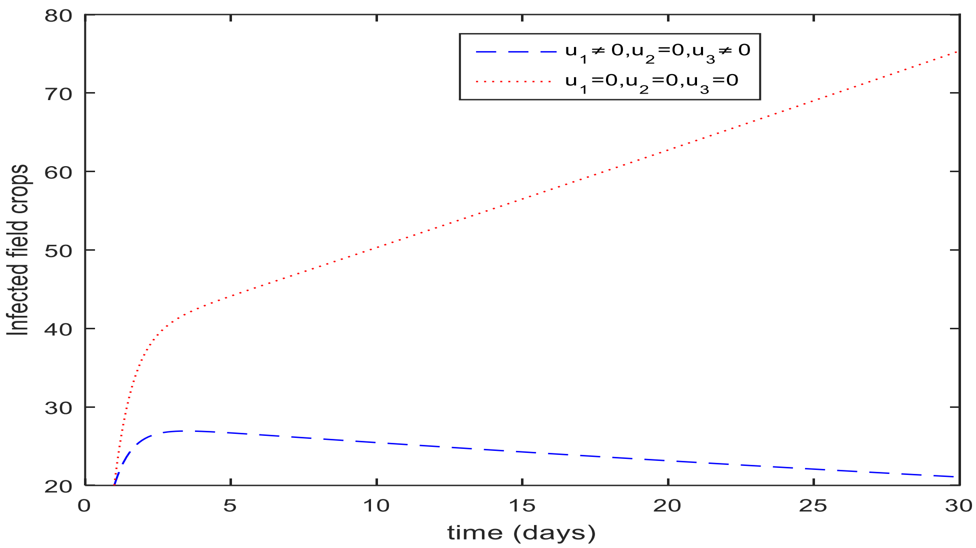

Figure 5: Simulation of infected crops with prevention measure (u_1)≠ 0 and removal of infected crops (u_3) ≠0

Strategy F: Prevention measure (u_1)= 0, Pesticide control (u_2) ≠ 0, removal of infected crops (u_3) ≠0

The use of pesticide and removal of infested crops will reduce the number of infested crops to 14 by 30days as shown in figure 6.

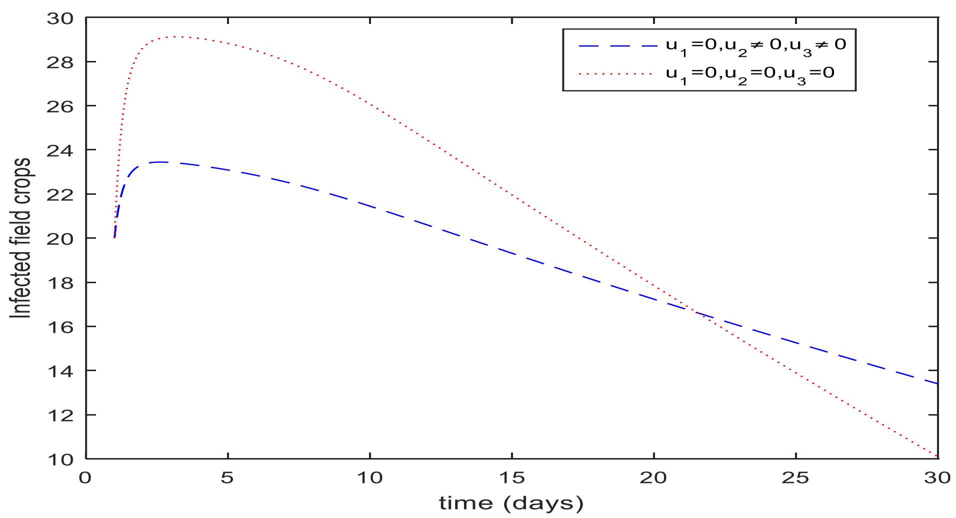

Figure 6: Simulation of infected crops with pesticide control (u_2) ≠ 0 and removal of infected crops (u_3) ≠0

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5. Conclusion

In this study, a logistic model with control measure for field crops was formulated and analysed to find the best measure for controlling pest infestation. We derived and analysed the model to obtain the condition for pest free equilibrium and pest endemic equilibrium. For minimizing pest infestation on field crop, control measures such as Prevention (u_1), Pest control (u_2) and removing infected (u_3) crops were used. In Section 4, we numerically analysed the control measures by considering the different integrated measures. It shows that each approach has the power to manage pest infestation, which is correlated with a similar result for tomatoes and cassava crops (Hugo et al., 2019). From the pair wise result, the integrated pest control measure called pesticide and removing of infested crops was observed to reduce the number of infected plants to number lower than other measures. The numerical results clearly directs that each integrated control measure strategy have a power to combat pest infestation, however, the combination of pesticide and removal of infected crops gave a better result.

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