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"IMPACT OF CORRUPT IMMIGRANT ON THE DYNAMICS OF CORRUPTION IN HUMAN POPULATION: A MATHEMATICAL MODELLING APPROACH"

IMPACT OF CORRUPT IMMIGRANT ON THE DYNAMICS OF CORRUPTION IN HUMAN POPULATION: A MATHEMATICAL MODELLING APPROACH

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Abstract

This study formulated and analyzed a nonlinear deterministic compartmental model of corruption dynamics that explicitly incorporates the influence of corrupt immigrants on a naive population. The population is divided into six compartments: susceptible, exposed, corrupt, prosecuted, jailed, and honest individuals. The model is shown to be well posed, with a positively invariant and attracting feasible region and non-negative solutions for all time. The corruption reproduction number, $\mathcal{R}_{\text{cp}}$, is derived using the next-generation matrix approach. The corruption-free equilibrium is proved locally and globally asymptotically stable when $\mathcal{R}_{\text{cp}} < 1$, while the corruption-present equilibrium is proved globally asymptotically stable when $\mathcal{R}_{\text{cp}} > 1$ using quadratic Lyapunov function. Sensitivity analysis identifies the recruitment, influence, prosecution, and reform rates as the parameters to which $\mathcal{R}_{\text{cp}}$ is most responsive, with reform exerting the strongest negative effect. Numerical simulations confirm the analytical results and show that the long-run endemic level is independent of the initial conditions. Although the sensitivity index of the corrupt-immigrant parameter is small (+0.0437), the simulations show that corrupt immigrants act as a persistent external source of corruption; neglecting this inflow would understate persistence corruption. The results support a combined strategy weighted toward reform, since rehabilitation, civic education, and moral recovery reduce both peak and endemic corruption and ease the burden on the prison system.

Keywords: Corruption; Mathematical modelling; Immigrant; Reproduction number; Stability; Sensitivity analysis

1. Introduction

Corruption is among the most intractable and persistent problems observed globally. It is not merely an abstract issue confined to academic discourse or news reportage; it is a daily reality that permeates almost every facet of life. Corruption undermines economic development and human associations [1], and it constitutes a crime against the poor, entailing the conversion of another person's property to personal use ([2, 3, 4]). It may take the form of non-violent activity aimed at deriving illicit gain from unsuspecting members of an association, community, or country at large [5]. It is a major cause of poverty worldwide, most especially in Africa [6], and a major obstacle to the rule of law, human rights, equity, and social justice ([7, 8]). Although most countries have established anti-corruption agencies, corruption remains endemic in many societies, and Nigeria, as a developing country, is a particularly salient case.

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Mathematical modelling provides an important tool for studying, understanding, and analyzing corruption dynamics in human populations, and for informing decisions regarding intervention strategies. Several authors have studied corruption dynamics using compartmental models. Akanni *et al* [2] proposed a model for financial crime dynamics with optimal control and cost-effectiveness strategies, concluding that strict law enforcement and intensified public enlightenment campaigns are required to stem the tide of financially corrupt individuals; however, their model did not consider the impact of corrupt immigrants or the exposure of a naive population. Gutema [9] and Alemneh [10] examined the effects of mass education and law enforcement, reporting that media campaigns and the imprisonment of corrupt individuals reduce exposed and corrupt populations. Fantaye and Birhanu [11] developed a model with optimal control measures that incorporates the influence of social influence on honest individuals. Jose *et al* [12] argued that corruption spreads in a society like an infectious disease and that preventive and therapeutic efforts should be maximized, while Assanga *et al* [13] explored the rehabilitation of corrupt individuals as a means of reducing the negative effects of corruption.

A further strand of the literature addresses the movement of individuals between corrupt and non-corrupt states. Nathan and Jacob [14] formulated a nonlinear model for corruption dynamics in Kenya and concluded that corruption will continue to undermine good governance, distort history, and public legislation. Rwat *et al* [15] examined the reintegration of formerly corrupt individuals into society, while Awadallah modelled corruption dynamics using fractal-fractional operators. Idisi *et al* [16] developed a model of crime and corruption dynamics with a cross-national focus, applied primarily to Guatemala, Venezuela, and Colombia, and reported that increased prosecution and sentencing reduce the prevalence of corruption. Collectively, these studies establish that corruption can be modelled as a transmission process and that enforcement, education, and rehabilitation are effective levers of control.

However, a common limitation runs through this body of work: the population is treated as closed. Susceptible individuals are recruited from within the system, but no allowance is made for corrupt individuals entering from outside it. This omission is consequential. If corrupt immigrants constitute an external source of corruption, then a closed-population model will systematically understate the reproduction number, overstate the effectiveness of internal interventions, and misidentify the threshold that reform and prosecution must overcome. Despite the significant and valuable findings of the existing literature, the role of corrupt immigrants in ethical systems has received little attention. This study therefore formulates a nonlinear deterministic compartmental model of corruption dynamics that explicitly incorporates the influence of corrupt immigrants on a naive population, derives the corruption reproduction number, establishes the local and global stability of the corruption-free and corruption-present equilibria, and quantifies through sensitivity analysis and numerical simulation the relative contribution of corrupt immigration and of the principal control measures. The principal finding is that corrupt immigrants act as a persistent

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external source of corruption that raises the reproduction number, though reform remains the most effective single intervention.

The study is organized as follows. Section 2 presents the formulation of the model. Section 3 provides the analysis of the model, including the invariant region, non-negativity of solutions, equilibrium points, stability analysis, and sensitivity analysis. Section 4 presents the numerical simulation, and Section 5 provides the conclusion.

2. Model Formulation

A non-linear deterministic compartmental mathematical model describing the dynamics of corruption in a population in the presence of corrupt immigrants is formulated. The total population at time t, denoted by $N(t)$, is subdivided into six distinct compartments based on individuals' status. The population classes are: Susceptible individuals $S(t)$, Exposed individuals $E(t)$, Corrupt individuals $C(t)$, Prosecuted individuals $P(t)$, Jailed individuals $J(t)$, and Honest individuals $H(t)$. Thus, the total population is given by

$$N(t) = S(t) + E(t) + C(t) + P(t) + J(t) + H(t). \quad (1)$$

The susceptible population grows at a rate of π due to recruitment into the system. However, corruption can enter the system through corrupt immigrants at a proportion of d. After interacting with corrupt individuals, susceptible individuals become exposed at a rate of βSCp; they can also become honest at a rate ε and they are subject to natural removal at a rate μ. Therefore, the rate of change of susceptible population is given by

$$\frac{dS}{dt} = \pi(1 - dC_p) - \beta SC_p - (\varepsilon + \mu)S \quad (2)$$

Similarly, exposed population grows when susceptible individuals interact with corrupt population at a rate βSC_p. The population reduces when some individuals progress to the corrupt class at a rate α. Further reduction takes place when some are reformed at a rate ε, while natural removal occurs at a rate μ. Then, the rate of change of exposed population is given by

$$\frac{dE}{dt} = \beta SC_p - (\alpha + \mu + \varepsilon)E \quad (3)$$

Corrupt population grows due to the influx of corrupt immigrant at a rate πdC_p. The population further increases as a result of progression from exposed class at a rate α and relapse at a rate $\omega(1 - \varepsilon)J_a$. Owing to prosecution, reform and natural removal, the population further reduced at the rates γ, ε and μ. Additionally, the population surged when a fraction of prosecuted return to corrupt practices at a rate τqP_r. Thus, the rate of change of corrupt population is given as

$$\frac{dC_p}{dt} = \pi dC_p + \alpha E + \omega(1 - \varepsilon)J_a - (\gamma + \varepsilon + \mu)C_p + \tau qP_r \quad (4)$$

Prosecution arise when corrupt individuals are nabbed and convicted at a rate γC_p. These individuals may be discharged from prosecution at a rate q and jailed at a rate σ. The population further decreased due to natural removal at a rate μ. Therefore, the dynamics of prosecuted population is given as

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$$\frac{dP_r}{dt} = \gamma C_p - (q + \sigma + \mu)P_r \quad (5)$$

Prosecuted individuals found guilty are jailed at a rate σ. The population reduces when they regain freedom at a rate ω and natural removal at a rate μ. Then, the rate of change of Jailed population is given by

$$\frac{dJ_a}{dt} = \sigma P - (\omega + \mu)J_a \quad (6)$$

When susceptible, exposed, and corrupt individuals are reformed, the population of honest individuals increases at a rate ε. A fraction of prosecuted individuals got reformed and increased the population at a rate $(1 - \tau)$. Further, natural removal rate ε reduced the population. Thus, the rate of change of honest population is given by

$$\frac{dH}{dt} = (1 - \tau)qP_r + \varepsilon(S + E + C_p + \omega J_a) - \mu H \quad (7)$$

Consequently, combining all the equations together, the mathematical model that describe the dynamics of corruption in human population is given by the following system of nonlinear ordinary differential equations:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi(1 - dc_p) - \beta S C_p - (\varepsilon + \mu)S \\
\frac{dE}{dt} &= \beta S C_p - (\alpha + \mu + \varepsilon)E \\
\frac{dC_p}{dt} &= \pi d C_p + \propto E + \omega(1 - \varepsilon)J_a - (\gamma + \varepsilon + \mu)C_p + \tau q P_r \\
\frac{dP_r}{dt} &= \gamma c_p - (q + \sigma + \mu)P_r \\
\frac{dJ_a}{dt} &= \sigma P_r - (\omega + \mu)J_a \\
\frac{dH}{dt} &= (1 - \tau)qP_r + \varepsilon(S + E + C_p + \omega J_a) - \mu H
\end{aligned}\right\} \quad (8)$$

Subject to the initial conditions

$S(0) = S_0, E(0) = E_0, C_p(0) = C_{p0}, P_r(0) = P_{r0}, J_a(0) = J_{a0}, H(0) = H_0.$

The flow diagram of the model incorporating corrupt immigrants is presented in Figure 1. The description of the model variables and parameters are also presented in Tables 1 and 2, respectively.

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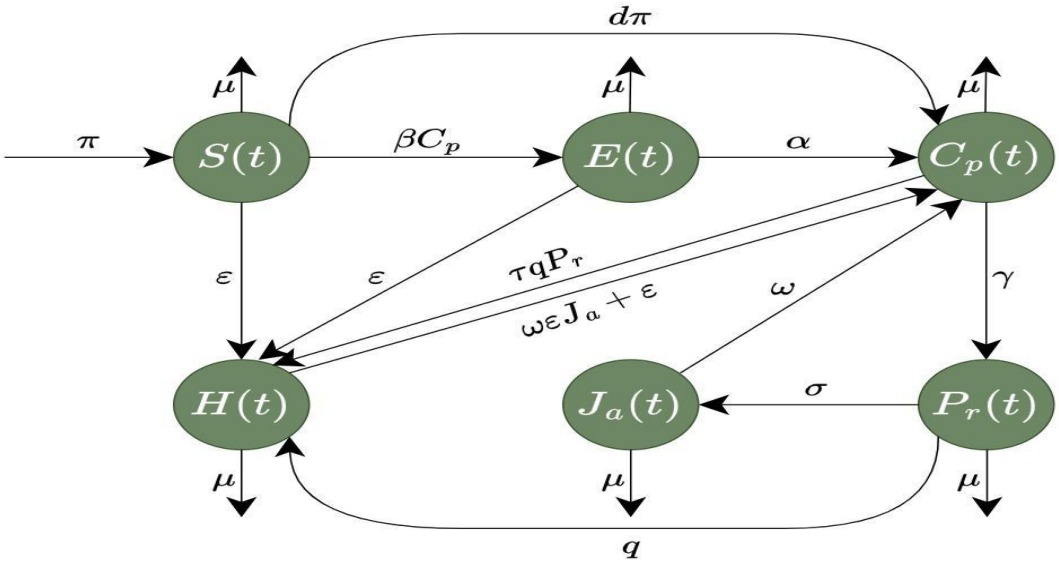


Figure 1: Schematic diagram of the corruption dynamics model

Table 1: Description of variables of the model

Variable	Description
$S(t)$	Susceptible population
$E(t)$	Exposed population
$C_p(t)$	Corrupt population
$P_r(t)$	Prosecuted population
$J_a(t)$	Jailed population
$H(t)$	Honest population

Table 2: Description of parameters of the corruption model

Parameter	Description
π	Recruitment rate
β	Influence rate
σ	Progression from
μ	Removal rate
ε	Reformed rate
q	Discharge rate
τ	Fraction of prosecuted individuals returning to corruption
d	Corrupt immigrant rate
ω	Freedom rate
α	Progression rate of exposed individuals to corrupt compartments

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γ	Prosecution rate of corrupt population

3. Model Analysis

3.1 The invariant region

Theorem 1: Let (S, E, C_p, P_r, J_a, H) be the solution of equation (8) with initial condition in a feasible region D, then $D \subset R_+^6$ is positively invariant.

Proof: Since the total population, N(t) = S(t) + E(t) + C_p(t) +P_r(t) + $J_a(t)$ + H(t), then the time derivative of $N(t)$ is given as

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dC_p}{dt} + \frac{dP_r}{dt} + \frac{dJ_a}{dt} + \frac{dH}{dt} \tag{9}$$

Simplifying Equation (9) gives

$$\frac{dN}{dt} = \pi - \mu(S + E + C_p + P_r + J_a + H) \tag{10}$$

So that

$$\frac{dN}{dt} \leq \pi - \mu N \tag{11}$$

Then, solving Equation (11) using the standard procedure yields

$$N(t) \leq N(0)e^{-\mu t} + \frac{\pi}{\mu}(1 - e^{-\mu t}) \tag{12}$$

Taking the limit as $t \to \infty$, then $N(t) \leq \frac{\pi}{\mu}$. This means that $\frac{\pi}{\mu}$ is the supremum of the total human population $N(t)$. If $N(0) \leq \frac{\pi}{\mu}$, then $N(t) \leq \frac{\pi}{\mu}$. Therefore, the region D is positively invariant. Furthermore, if $N(0) \geq \frac{\pi}{\mu}$, then the solution enters D in finite time as $N(t)$ approaches $\frac{\pi}{\mu}$ asymptotically. This implies that the feasible region is attracting. Mathematically, this means that all solutions eventually enter D.

3.2 Non-negativity of solutions

Theorem 2: The solutions of the corruption model given by {S, E, C_p, P_r, J_a, H} with non-negative initial conditions ($S(0)$, $E(0)$, $C_p(0)$, $P_r(0)$, $J_a(0)$, $H(0)$) remain non-negative for all time $t > 0$.

Proof: The proof is based on the approach employed in ([17, 18, 19]). Then, it follows from the nonlinear model that

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$$\left.\frac{dS}{dt}\right|_{S=0} = \pi(1 - dC_p) > 0$$

$$\left.\frac{dE}{dt}\right|_{E=0} = \beta S C_p \geq 0$$

$$\left.\frac{dC_p}{dt}\right|_{C_p=0} = \alpha E + \omega(1-\varepsilon)J_a + \tau q P_r \geq 0 \qquad (12)$$

$$\left.\frac{dP_r}{dt}\right|_{P_r=0} = \gamma C_p \geq 0$$

$$\left.\frac{dJ_a}{dt}\right|_{J_a=0} = \sigma P_r \geq 0$$

$$\left.\frac{dH}{dt}\right|_{H=0} = (1-\tau)qP_r + \varepsilon(S + C_p + E + \omega J_a) \geq 0$$

It is straightforward from (11) that the rates of change of model (8) are non-negative in the bound R_+^6. This implies that the solutions of the corruption model are non-negative for all time $t > 0$.

3.3 Stability Analysis and Steady States of the Model

3.3.1 Corruption-free equilibrium (CFE)

The corruption-free equilibrium point is a steady state where there is no corruption in the society or community. This implies that all corruption related variables $E = C_p = P_r = J_a = 0$. Then the resulting corruption-free steady state denoted $\mathcal{E}^*$ is given by

$$\mathcal{E}_0 = \left(\frac{\pi}{\varepsilon+\mu}, 0, 0, 0, 0, \frac{\varepsilon\pi}{\mu(\varepsilon+\mu)}\right) \qquad (13)$$

The corruption reproduction number, which is a crucial threshold in determining the CFE steady state is next considered. It is the parameter used to obtain the conditions under which incidence of corruption will persist or be curbed in the population. This is achieved by employing the next-generation matrix approach, ([20, 21, 22]) characterized by the spectral radius of the product of FV^{-1}, given by

$$\mathcal{R}_{cp} = \rho F V^{-1},$$

where F represents the new infection matrix (transmission matrix) and V represents the transition matrix are obtained as

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$$F = \begin{pmatrix} 0 & \frac{\beta\pi}{k_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

and

$$V = \begin{pmatrix} k_2 & 0 & 0 & 0 \\ -\alpha & k_3 - \pi d & -\tau q & -\omega(1-\varepsilon) \\ 0 & -\gamma & k_4 & 0 \\ 0 & 0 & -\sigma & k_5 \end{pmatrix} \tag{15}$$

where $k_1 = (\varepsilon + \mu)$, $k_2 = (\alpha + \mu + \varepsilon)$, $k_3 = (\gamma + \varepsilon + \mu)$, $k_4 = (q + \sigma + \mu)$ and $k_5 = (\omega + \mu)$. Therefore, the corruption reproduction number of the corruption model (8) in the presence of corrupt immigrant, represented by $\mathcal{R}_0$, is the dominant eigenvalue of the spectral radius of ρFV^{-1} obtained as

$$\mathcal{R}_{cp} = \frac{\alpha\beta\pi k_4 k_5}{k_1 k_2 [k_4 k_5 (k_3 - \pi d) - \tau q \gamma k_5 - \sigma\omega(1-\varepsilon)]} \tag{16}$$

It is noteworthy to state that the parametric conditions $(k_3 - \pi d) > 0$ and $\left(k_4 k_5 - \tau q \gamma k_5 - \sigma\omega(1-\varepsilon)\right) > 0$ hold, so that $\mathcal{R}_{cp}$ is positive.

Theorem 3: The corruption-free state of the corruption system (8) is locally asymptotically stable provided the associated threshold parameter $\mathcal{R}_{cp}$ is less than unity.

Proof: Linearizing system (8)**,** the Jacobian matrix of the corruption model (8) evaluated at corruption-free equilibrium $\mathcal{E}_0$ is obtained as

$$J_{CFE} = \begin{pmatrix} -k_1 & 0 & -\pi d - \frac{\beta\pi}{k_1} & 0 & 0 & 0 \\ 0 & -k_2 & \frac{\beta\pi}{k_1} & 0 & 0 & 0 \\ 0 & \alpha & \pi d - k_3 & \tau q & \omega(1-\varepsilon) & 0 \\ 0 & 0 & \gamma & -k_4 & 0 & 0 \\ 0 & 0 & 0 & \sigma & -k_5 & 0 \\ \varepsilon & \varepsilon & \varepsilon & (1-\tau)q & \varepsilon\omega & -\mu \end{pmatrix} \tag{17}$$

Using the characteristic equation $|\Im_{CFE} - \lambda I| = 0$, two of the eigenvalues are obtained as $\lambda_1 = -k_1$ and $\lambda_2 = -\mu$, and the remaining eigenvalues are obtained from the quartic equation

$$\lambda^4 + P_1\lambda^3 + P_2\lambda^2 + P_3\lambda + P_4 = 0 \tag{18}$$

where

$P_1 = k_2 + k_3 + k_4 + k_5 > 0$

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$$P_2 = k_2k_3 + k_2k_4 + k_2k_5 + k_3k_4 + k_4k_5 + k_3k_5 - \pi d\left(k_2 + k_4 + k_5\right) - \tau q\gamma - \frac{\beta\pi\alpha}{k_1} > 0$$

$$P_3 = k_2k_3k_4 + k_2k_3k_5 + k_2k_4k_5 + k_3k_4k_5 - \pi d(k_2k_4 + k_2k_5 + k_4k_5) - \tau q\gamma(k_2 + k_5) - \gamma\sigma\,\omega(1 - \varepsilon) - \frac{\beta\pi\alpha k_4k_5}{k_1} > 0$$

$$P_4 = \ k_2[k_3k_4k_5 - \pi dk_4k_5 - \tau q\gamma k_5 - \gamma\sigma\,\omega(1 - \varepsilon)](1 - \mathcal{R}_{cp})$$

Clearly, coefficient P_1 of the quartic polynomial (18) is positive, coefficients P_2 and P_3 will be positive provided the positive terms are greater than the negative terms, and coefficient P_4 is also positive provided $\mathcal{R}_{cp} < 1$. Then, by Descarte's rule of signs, Equation (18) has four positive roots indicating that the four eigenvalues are negative, real and distinct. Hence, the corruption-free equilibrium point of system (8) is locally asymptotically stable when $\mathcal{R}_{cp} < 1$. This implies that corruption-free society can be achieved if the threshold parameter $\mathcal{R}_{cp}$ is maximally maintained below unity.

3.3.2 Corruption-present equilibrium

Corruption-present equilibrium refers to a steady state solution where there is presence of solution in the population. It is determined to provide the long-term effect of corruption dynamics in the population. Let the corruption-present equilibrium point of system (8) be denoted by $\mathcal{E}^{**} = (S^{**}, E^{**}, C_p^{**}, P_r^{**}, J_a^{**}, H^{**})$, $M = \omega(1 - \varepsilon)$, $(1 - \tau)q$ and $\lambda^{**} = \beta C_p^{**}$ be the force of influence. Then, solving (8) simultaneously gives

$$\mathrm{S}^{**} = \frac{\pi\,[k_2\mathrm{k}_4\mathrm{k}_5(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})-k_2\mathrm{k}_5\gamma\tau\mathrm{q}(k_1+\lambda^{**})]-k_2\sigma M\gamma\,(K_1+\lambda^{**})]}{(\mathrm{k}_1+\lambda^{**})[k_4\mathrm{k}_5\{k_2(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})+\propto\pi\lambda^{**}\}-k_2k_5\gamma\tau\mathrm{q}\,(k_1+\lambda^{**})-k_2\sigma M\gamma\,(k_1+\lambda^{**})}$$

$$E^{**} = \frac{\pi\lambda^{**}\mathrm{k}_5[k_2\mathrm{k}_4(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})-k_2\gamma\tau\mathrm{q}(k_1+\lambda^{**})]-k_2\sigma M\gamma\,(k_1+\lambda^{**})]}{\mathrm{k}_2(\mathrm{k}_1+\lambda^{**})[k_4\mathrm{k}_5\{k_2(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})+\propto\pi\lambda^{**}\}-k_2k_5\gamma\tau\mathrm{q}\,(k_1+\lambda^{**})-k_2\sigma M\gamma\,(k_1+\lambda^{**})}$$

$$\mathrm{C}_p{}^{**} = \frac{k_4k_5\propto\pi\lambda^{**}}{\mathrm{k}_5[\mathrm{k}_4(k_2(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})+\propto\pi\mathrm{d}\lambda^{**})-k_2\gamma\tau\mathrm{q}(k_1+\lambda^{**})]-k_2\sigma M\gamma\,(k_1+\lambda^{**})} \qquad (18)$$

$$\mathrm{P}_r{}^{*} = \frac{k_5\propto\sigma\gamma\pi\lambda^{**}}{\mathrm{k}_5[\mathrm{k}_4(k_2(k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})+\propto\pi\mathrm{d}\lambda^{**})-k_2\gamma\tau\mathrm{q}(k_1+\lambda^{**})]-k_2\sigma M\gamma\,(k_1+\lambda^{**})}$$

$$\mathrm{J}_a{}^{**} = \frac{\propto\sigma\gamma\pi\lambda^{**}}{\mathrm{k}_5[\mathrm{k}_4((k_1+\lambda^{**})(\mathrm{k}_3-\pi\mathrm{d})+\lambda^{**}\propto\pi\mathrm{d})-\gamma k_2\tau\mathrm{q}(k_1+\lambda^{**})]-k_2\sigma M\gamma\,(k_1+\lambda^{**})}$$

Substituting C_p^{**} in (18) into the force of influence yields

$$\lambda^{**} = \frac{\mathrm{k}_1\mathrm{k}_2\{\mathrm{k}_4\mathrm{k}_5(\mathrm{k}_3-\pi\mathrm{d})-\ k_5\gamma\tau\mathrm{q}-\sigma\mathrm{M}\gamma\,(\mathrm{R}_0-1)}{\mathrm{k}_2\mathrm{k}_4\mathrm{k}_5(\mathrm{k}_3-\pi\mathrm{d})+\ \mathrm{k}_4\mathrm{k}_5\propto\pi\mathrm{d}-k_2k_5\ \gamma\tau\mathrm{q}-\ k_2\sigma M\gamma} \qquad (19)$$

Hence, corruption will persist in the population as long as the threshold parameter $\mathcal{R}_{cp}$ exceeds unity. This implies that corruption will be difficult to control in the population.

3.3.3 Global stability of corruption-free equilibrium

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The global asymptotic stability of the corruption-free equilibrium of system (8) is carried out using the approach applied in ([23,24]). Then, the corruption model (8) can be restructured in compact form as

$$\begin{aligned} \frac{dX}{dt} &= F(X,Y) \\ \frac{dY}{dt} &= G(X,Y), G(X,0) = 0 \end{aligned} \tag{20}$$

where $X\epsilon {R_+}^2$ and $Y\epsilon {R_+}^4$. X component represents the uncorrupt compartments while, Y represents the corrupt immigrants. Given that the corruption-free equilibrium of system (8) is $\mathcal{E}_0 = (X^*, 0)$, then the following properties must be satisfied in order to establish the global dynamics of system (8)

H_1: for $\frac{dX}{dt} = F(X^*, 0)$, X^* is globally asymptotically stable

H_2: $G(X,Y) = AY - \hat{\mathrm{G}}(X,Y), \hat{\mathrm{G}}(X,Y) \geq 0$ for $(X,Y) \in D$

$A = \frac{\partial G}{\partial Y}$ is a matrix evaluated $(X^*, 0)$, where the off diagonal elements are non-negative

Theorem 4: *The corrupt-free equilibrium of the system (3.8) is globally asymptotically stable if the basic reproduction number is less than one and properties H_1 and H_2 are satisfied.*

Proof: $F(X,Y)$ and $G(X,Y)$ are obtained from Equation (20) as

$$F(X,Y) = \begin{pmatrix} \pi(1 - dC_p) - \beta S C_p - (\varepsilon + \mu)S \\ (1-\tau)qP_r + \varepsilon(S + C_p + E + \omega J_a) - \mu H \end{pmatrix} \tag{21}$$

and

$$G(X,Y) = \begin{pmatrix} \beta S C_p - (\alpha + \mu + \varepsilon)E \\ \pi d C_p + \alpha E + \omega(1-\varepsilon)J_a - (\gamma + \varepsilon + \mu)C_p + \tau q P_r \\ \gamma - (q + \sigma + \mu)P_r \\ \sigma P_r - (\omega + \mu)J_a \end{pmatrix} \tag{22}$$

Then, $\frac{dX}{dt} = F(X,0)$ Implies

$$\begin{aligned} \frac{dS}{dt} &= \pi - (\varepsilon + \mu)S \\ \frac{dH}{dt} &= \varepsilon S - \mu H \end{aligned} \tag{23}$$

Solving Equation (23) simultaneously at corruption-free equilibrium using the standard technique yields

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$$S(t) = S(0)e^{-(\mu+\varepsilon)t} + \frac{\pi}{\mu+\varepsilon}\left(1 - e^{-(\mu+\varepsilon)t}\right)$$
$$H(t) = H(0)e^{-(\mu+\varepsilon)t} + \varepsilon\left(S(0)e^{-(\mu+\varepsilon)t} + \frac{\pi}{\mu+\varepsilon}\left(1 - e^{-(\mu+\varepsilon)t}\right)\right) \quad (24)$$

Hence, $S(t), H(t) \to \left(\frac{\pi}{\varepsilon+\mu}, \frac{\varepsilon\pi}{\varepsilon+\mu}\right)$ irrespective of the initial value of the variable at $t \to \infty$. Therefore $X^*$is globally asymptotically stable satisfying H_1. Furthermore, the matrix A with non-negative off-diagonal elements is obtained as

$$A = \frac{\partial G}{\partial X_2}(X^*, 0) = \begin{pmatrix} -(\alpha+\mu) & \beta S^0 & 0 & 0 \\ \alpha & \pi d - (\gamma+\varepsilon+\mu) & \tau q & \omega(1-\varepsilon) \\ 0 & \gamma & -(q+\sigma+\mu) & 0 \\ 0 & 0 & \sigma & -(\omega+\mu) \end{pmatrix} \begin{pmatrix} E \\ C_p \\ P_r \\ J_a \end{pmatrix} \quad (25)$$

Then, it follows that $AY - \hat{\mathrm{G}}(X, Y)$ is obtained as

$$\hat{\mathrm{G}}(X_1, X_2) = \begin{pmatrix} \beta C_p(S^0 - S) \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (26)$$

Since $0 \le S \le \frac{\pi}{\varepsilon+\mu}$ and $0 \le H \le \frac{\varepsilon\pi}{\varepsilon+\mu}$ it is obvious that $\hat{\mathrm{G}}(X_1, X_2) \ge 0$. Thus property H_2 is satisfied. Hence, corruption-free equilibrium is globally asymptotically stable.

3.3.5 Global stability of corruption-present equilibrium

Here, the long-term effect of corruption-present steady state is carried out using approach adopted in ([19, 25, 26,])

***Theorem 5*:** The corruption-present steady state of the system (8) is globally asymptotically stable in D when $\mathcal{R}_{cp} > 1$.

Proof: Let the quadratic Lyapunov function $\mathcal{H}: \{(S, E, C_p, P_r, J_a, H) \in D: S(t), E(t), C_p(t), P_r(t), J_a(t), H(t) > 0\} \to \mathbb{R}$ be defined by

$$\mathcal{H} = \frac{1}{2}\left\{(S - S^{**}) + (E - E^{**}) + \left(C_p - C_p^{**}\right) + (P_r - P_r^{**}) + (J_a - J_a^{**}) + (H - H^{**})\right\}^2 \quad (27)$$

Differentiating the continuous function $\mathcal{H}$ with respect to time gives

$$\frac{d\mathcal{H}}{dt} = \left\{(S - S^{**}) + (E - E^{**}) + \left(C_p - C_p^{**}\right) + (P_r - P_r^{**}) + (J_a - J_a^{**}) + (H - H^{**})\right\} \times \frac{d}{dt}(S + E + C_p + P_r + J_a + H)$$
$$= \left\{(S - S^*) + (E - E^*) + \left(C_p - C_p^*\right) + (P_r - P_r^*) + (J_a - J_a^*) + (H - H^*)\right\} \times \left\{\pi - \mu(S + E + C_p + P_r + J_a + H)\right\}$$

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$$\leq (N - N^{**}) \times (\pi - \mu N) \tag{28}$$

Further simplification of Equation (28) yields

$$\frac{d\mathcal{H}}{dt} \leq -\mu(N - N^{**}) \times (N - \frac{\pi}{\mu}) \tag{29}$$

Since $N^{**} = \frac{\pi}{\mu}$, then Equation (28) becomes

$$\frac{d\mathcal{H}}{dt} \leq -\mu(N - N^{**})^2 \tag{30}$$

The derivative of the continuously differentiable function $\mathcal{H}$ is a Lyapunov function as $d\mathcal{H}/dt \leq 0$. Additionally, $\dot{\mathcal{H}} = 0$ provided $S = S^{**}$, $E = E^{**}$, $C_p = C_p^{**}$, $P_r = P_r^{**}$, $J_a = J_a^{**}$, $H = H^{**}$. Consequently, the largest invariant set on which the derivative of $\mathcal{H}$ vanishes is the singleton set $\{\mathcal{E}^{**}\}$. It follows, by LaSalle's invariance principle [27], that the corruption-present steady state is globally asymptotically stable. Hence, all solution trajectories converge to the unique endemic equilibrium, irrespective of the initial population sizes.

3.3.6 Sensitivity analysis

The sensitivity analysis of the parameters associated with the corruption reproduction number $\mathcal{R}_{cp}$ is performed in order to observe the changes that may occur in the dynamics of corruption. This is achieved using the normalized forward sensitivity index ([28, 29, 30]). The normalized forward sensitivity indices of the effective reproduction number with respect to its parameters are given as

$$\Psi_{\upsilon}^{\mathcal{R}_{cp}} = \frac{\partial \mathcal{R}_{cp}}{\partial \upsilon} \times \frac{\upsilon}{\mathcal{R}_{cp}} \tag{31}$$

where υ is the parameter of the corruption reproduction number. When (31) is applied, the sensitivity index of each parameter of $\mathcal{R}_{cp}$ is calculated with values of parameters given in Table 3, and the sensitivity indices are given in Table 4.

Table 3: Parameter values of the corruption model

Parameter	Value	Source
π	0.003	Assumed
β	0.85	[2]
μ	0.02	[2]
σ	0.67	[2]
q	0.33	[2]
ε	0.05	[2]
τ	0.65	[2]
ω	0.55	[2]
γ	0.2	[2]
d	0.15	Assumed

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α	0.40	Assumed

Table 4: Sensitivity indices corruption reproduction number $\mathcal{R}_{cp}$ relative to its parameters

Parameter	Value
π	+1.0437
β	+1.0000
μ	-0.4600
σ	+1.1300
q	-0.0980
ε	-1.2400
τ	+0.3900
ω	+0.0390
γ	-0.5300
d	+0.0437
α	+0.1489

4. Numerical Simulation

Here, numerical simulations are performed to corroborate and strengthen the qualitative analytical findings of the study using the parameter values given in Table 3, and the results are presented graphically.

Figure 2 presents a three-dimensional surface of the corruption reproduction number plotted against the influence rate, β, and the progression rate from the exposed class to the corrupt class, α. The surface rises as either parameter increases. β measures the strength with which a corrupt individual influences a susceptible individual, whereas α measures the speed at which an already exposed individual joins the corrupt class. An increase in either quantity generates more secondary cases from a single typical corrupt individual. This indicates that corruption is more likely to persist in a society where corrupt behaviour is readily copied and where individuals transition rapidly from first contact with corruption to active participation. Figure 3 illustrates the joint effect of the recruitment rate, π, and the progression rate, α. The surface increases with both parameters. A larger recruitment rate implies that more individuals enter the population over time. Some of these new members are available to be influenced, and a fraction may also enter already associated with corrupt inflows. When this is combined with a higher value of α, newly exposed individuals become corrupt more rapidly, so that $\mathcal{R}_{cp}$ becomes larger. The is consistent with the computed sensitivity index of π, which is greater than unity. Recruitment does not merely replace those who leave the system; it also enlarges the pool through which corruption can spread. Hence, a growing population, in the absence of a matching increase in reform or prosecution, favours the persistence of corruption.

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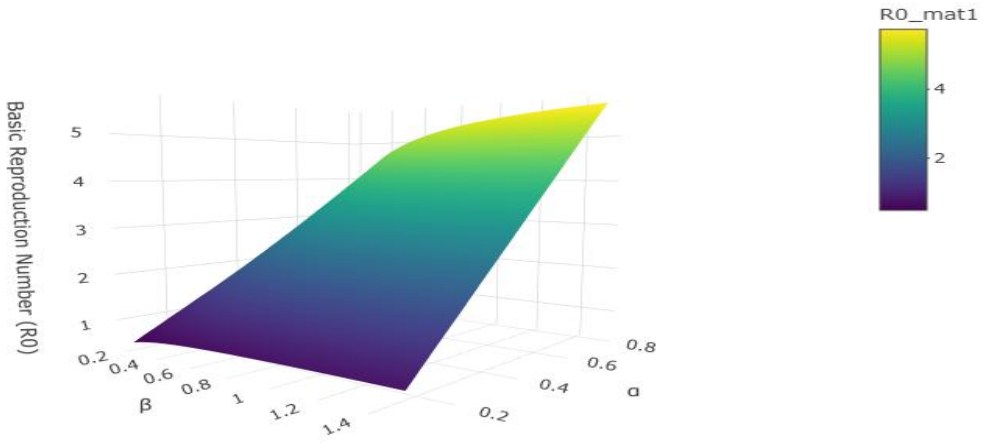


Figure 2: Impact of influence and progression rates on corruption reproduction number $\mathcal{R}_{cp}$

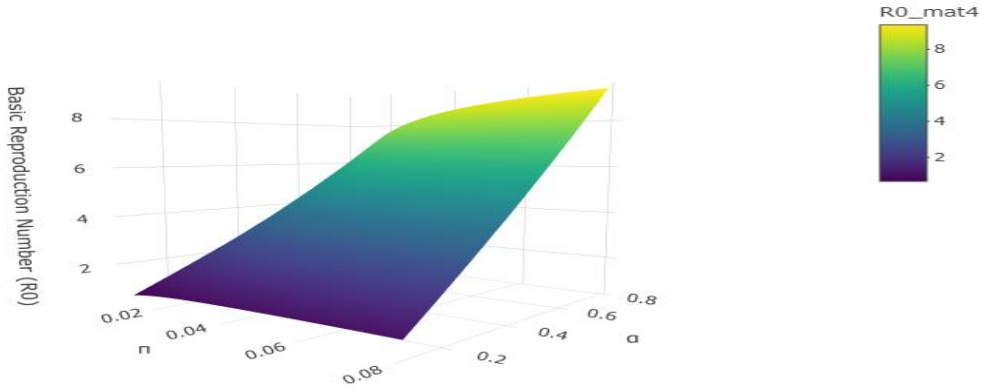


Figure 3: Influence of recruitment and progression rates on $\mathcal{R}_{cp}$

Figure 4 presents the surface most closely tied to the aim of this study. It plots $\mathcal{R}_{cp}$ against the influence rate, β, and the proportion of corrupt immigrants, d. The surface slopes upward in both directions. An increase in β produces a pronounced rise in $\mathcal{R}_{cp}$, whereas an increase in d produces a smaller but steady rise. Even when β is held fixed, a higher value of d elevates the entire surface. This indicates that corrupt immigrants act as an external source of corruption: they add new corrupt individuals to the population and help to prevent corruption from dying out. The effect of d is weaker than that of β, which is consistent with the sensitivity table: the index of β is +1.0000, whereas the index of d is only +0.0437. The policy implication nevertheless remains important. A society that ignores the effect of corrupt immigrant d would understate the level of $\mathcal{R}_{cp}$. Screening and reducing corrupt inflows will not, by itself, eradicate corruption, but it will lower the threshold that reform and prosecution must overcome.

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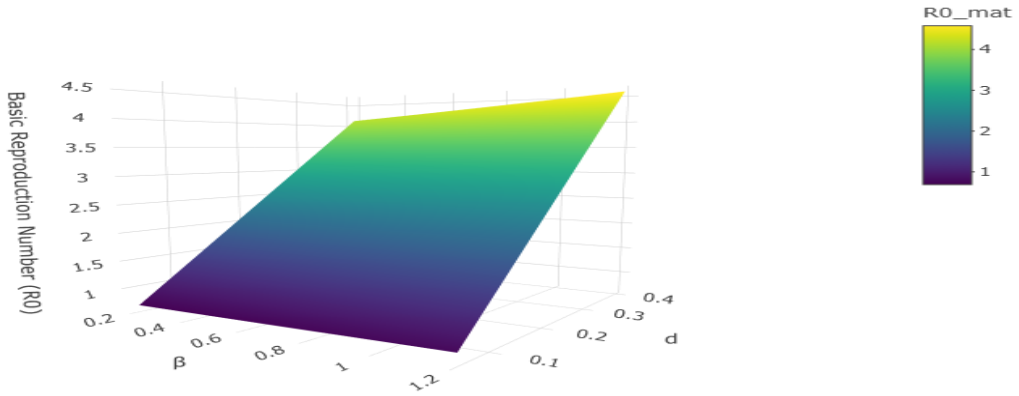


Figure 4: Sensitivity of $\mathcal{R}_{cp}$ to the parameters β and d

Figure 5 presents the two principal control parameters: the reform rate, ε, and the prosecution rate, γ. Unlike the preceding surfaces, this surface slopes downward. $\boldsymbol{\mathcal{R}_{cp}}$ declines as either parameter increases. The decline is considerably steeper with respect to ε. When the reform rate is very low, $\mathcal{R}_{cp}$ can become extremely large, even if prosecution is moderately high. When the reform rate is raised, $\mathcal{R}_{cp}$ falls rapidly and the surface becomes almost flat at low values. Reform removes individuals from the susceptible, exposed, and corrupt classes and moves them into the honest class, whereas prosecution removes individuals from the active corrupt class. Both reduce the number of secondary cases, but rehabilitation exerts the stronger effect. This therefore supports a combined strategy, with greater weight placed on reform.

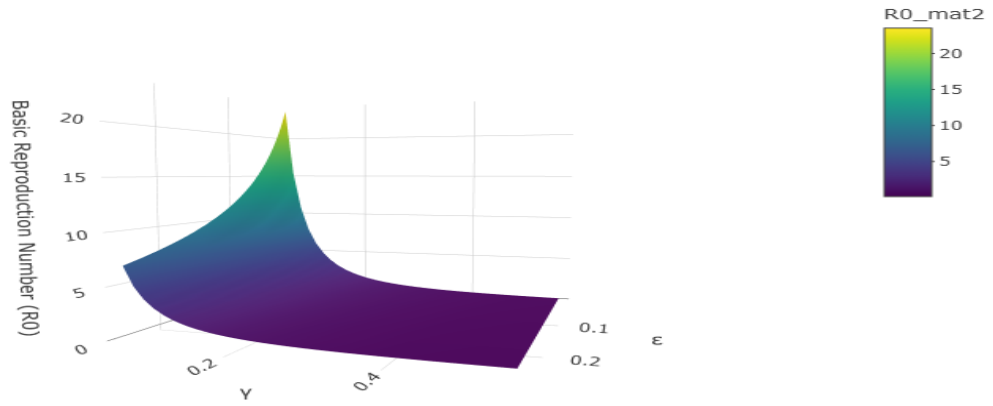


Figure 5: Effect of prosecution and reform rates on corruption reproduction number $\mathcal{R}_{cp}$.

Taken together, Figures 6 and Figure 7 illustrate two aspects of the same process: as the influence rate increases, the susceptible class contracts while the corrupt class expands. Figure 8 shows that susceptible class declines rapidly at the beginning for every value of ε. This early decline occurs because many susceptible individuals are exposed through contact with the corrupt class. After

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this initial decrease, the long-run level of the susceptible class depends on ε. When reform is weak ($\varepsilon = 0.02$), the susceptible class remains at its lowest. When reform is moderate ($\varepsilon = 0.10$), more individuals leave the path to corruption, and the susceptible class subsequently settles at a higher level. When $\varepsilon = 0.20$, corruption reduces significantly, because a very high reform rate also moves susceptible individuals directly into the honest class. The shows that reform does not simply save the susceptible class; it redistributes the population toward honesty. Figure 9 provides the clearest illustration of the benefit of reform. For $\varepsilon = 0.02$, the corrupt class rises to a high peak exceeding 0.40 of the population and subsequently remains above 0.20 in the long run. When $\varepsilon = 0.05$, the peak is lower and the endemic level is approximately 0.11. For $\varepsilon = 0.10$, both the peak and the long-run level fall further. For $\varepsilon = 0.20$, the corrupt class rises only slightly and then declines toward zero. This constitutes the numerical counterpart of the large negative sensitivity index of ε. A higher reform rate shortens the time spent in the corrupt class and converts more individuals before they generate new cases. In practical terms, if rehabilitation, civic education, and moral recovery are strengthened, corruption can be driven down and, at a sufficiently high reform rate, almost eliminated. The impact of the reform rate on the jailed population is depicted in Figure 10 When reform is weak, many individuals become corrupt, and a fraction of them are subsequently prosecuted and jailed. This produces a high peak and a high endemic level in the jailed class. When reform is strong, fewer individuals become corrupt in the first place, and

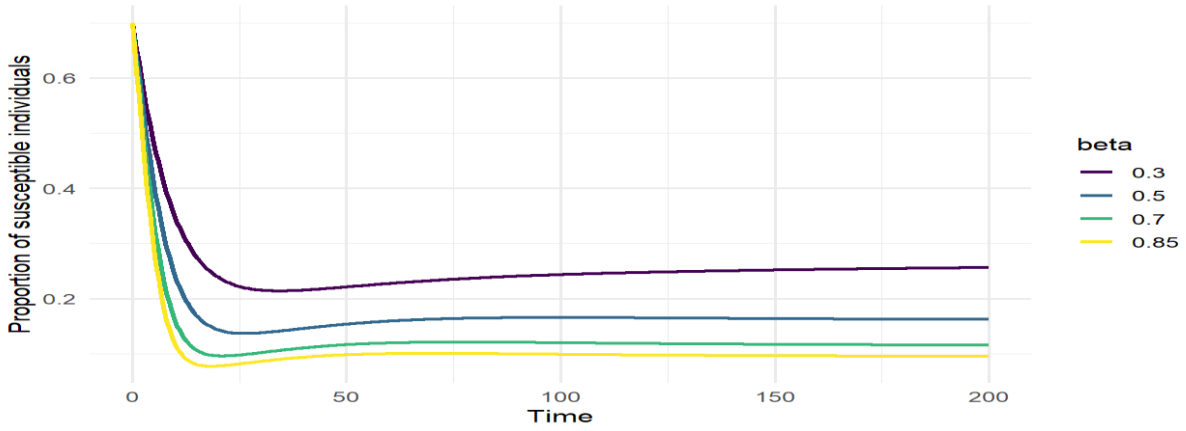


Figure 6: Impact of influence rate on the susceptible population

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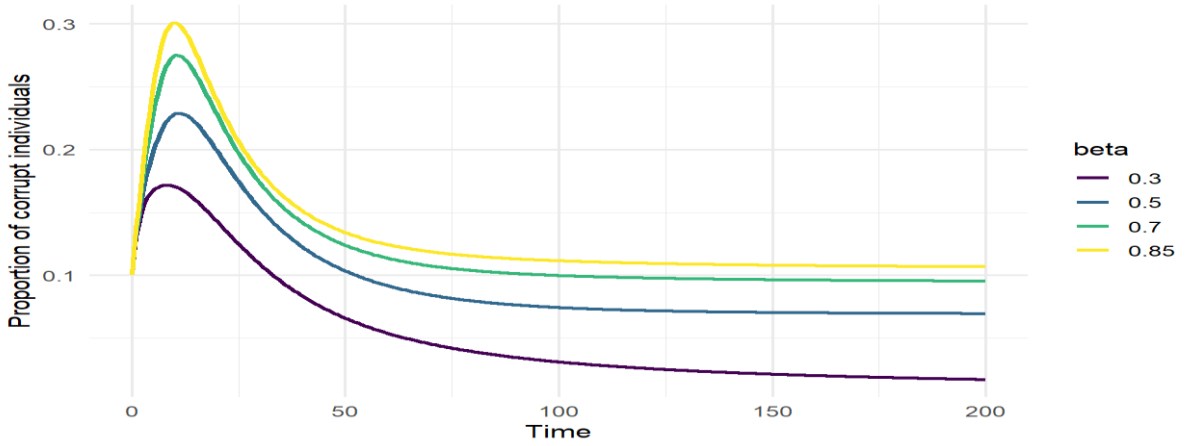


Figure 7: Impact of influence rate on the corrupt population.

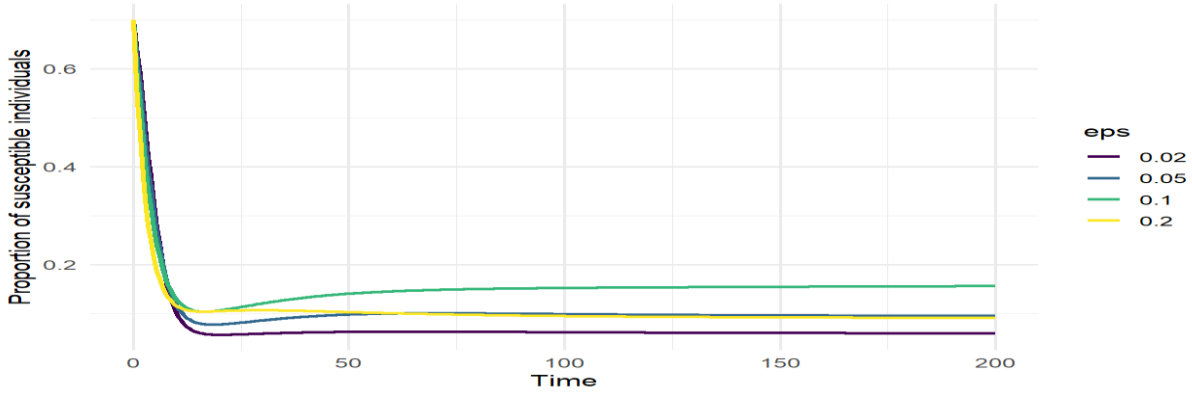


Figure 8: Impact of reform rate on the susceptible population

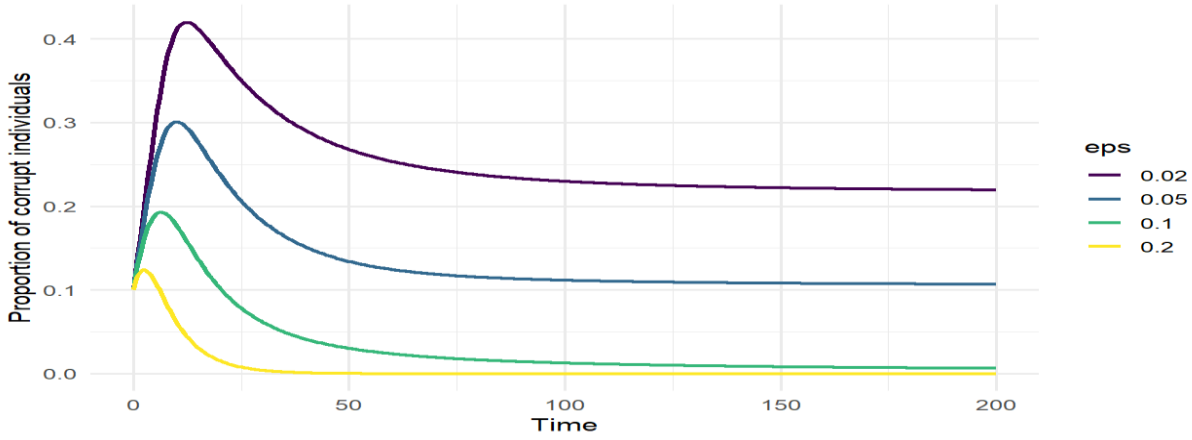


Figure 9: Effect of reform rate on the corrupt population

consequently fewer enter jail. At ε = 0.20, the jailed class declines almost immediately toward zero. The figure shows that reform reduces the need for imprisonment. Since jail is a downstream compartment, if individuals are converted before they become persistently corrupt, the prison load

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associated with corruption falls. Additionally, Figure 11 examines whether the endemic level depends on the starting point. Four initial conditions are used. The curves begin at different heights and attain different early peaks. IC-3 produces the highest peak, slightly above 0.36; IC-2 peaks near 0.32, IC-1 near 0.26, and IC-4 near 0.24. After approximately 40 days, the four trajectories coincide, settling at the same endemic level. The short-run outbreak may be larger or smaller depending on the initial data, but the long-run state remains unchanged. This supports the stability analysis: when $\mathcal{R}_{cp} > 1$, solutions neither wander nor return to the corruption-free state; they approach a single (unique) corruption-present equilibrium. This implies that corruption will persist in the population as long as $\mathcal{R}_{cp} > 1$ and it will be difficult to curb or control in the population.

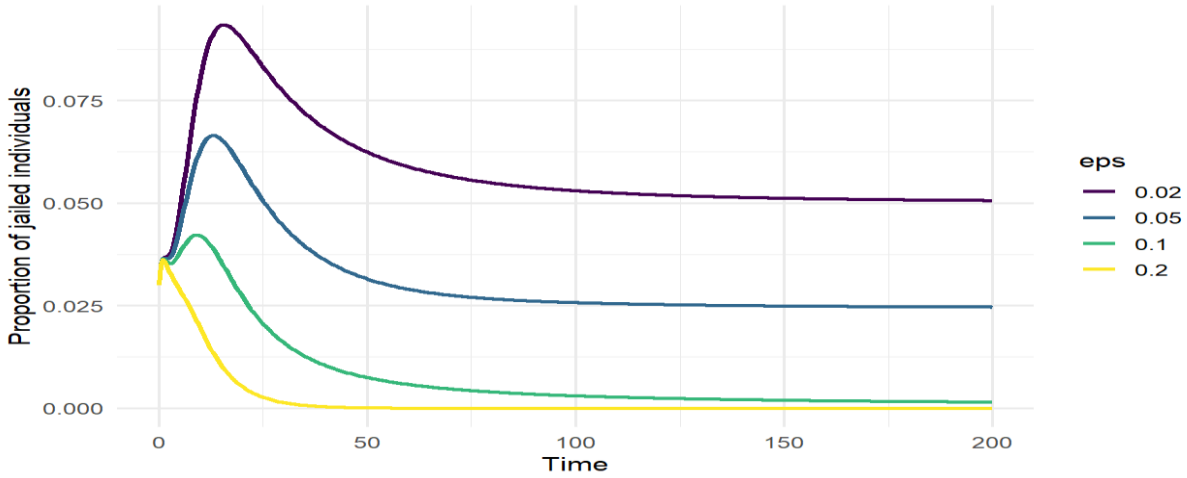


Figure 10: Influence of reform rate on the jailed population

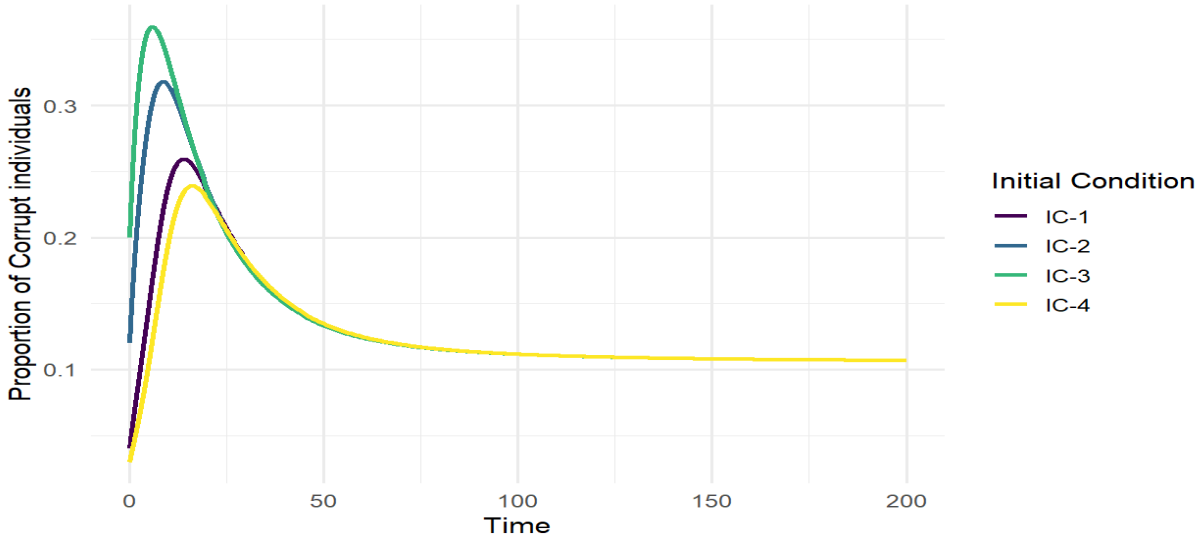


Figure 11: Convergence of global dynamics of corruption-present equilibrium from different initial data

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5. Conclusion

This study formulated and analyzed a nonlinear deterministic compartmental model of corruption dynamics that explicitly incorporates the influence of corrupt immigrants on a naive population. The model was shown to be well posed: the feasible region is positively invariant and attracting, and solutions remain non-negative for all time. The corruption-free equilibrium was proved locally and globally asymptotically stable when $\mathcal{R}_{cp} < 1$, while the corruption-present equilibrium was proved globally asymptotically stable when $\mathcal{R}_{cp} > 1$, using a Lyapunov function together with LaSalle's invariance principle. Numerical simulations corroborated these results, with trajectories from four distinct initial conditions converging to a unique endemic equilibrium, thereby confirming that the long-run prevalence of corruption is determined by the threshold parameter rather than by the initial population configuration. Sensitivity analysis identified the recruitment rate, influence rate, prosecution rate, and reform rate as the parameters to which $\mathcal{R}_{cp}$ is most responsive, with reform exerting the strongest negative effect. The three-dimensional surfaces were consistent with these indices: $\mathcal{R}_{cp}$ increased with the influence, progression, recruitment, and corrupt-immigrant parameters, but declined steeply with the reform rate and, to a lesser extent, with the prosecution rate.

The principal contribution of this study is the explicit incorporation of corrupt immigration. Although its sensitivity index was comparatively small (+0.0437), the corrupt-immigrant parameter elevated the entire reproduction surface and sustained the corrupt class, indicating that corrupt immigrants function as a persistent external source of corruption. A model that neglects this inflow would therefore understate the endemic level of corruption. This finding carries a clear policy implication: screening and reducing corrupt inflows, while insufficient in isolation, lower the threshold that reform and prosecution must overcome.

Collectively, the results support a combined intervention strategy weighted toward reform. Rehabilitation, civic education, and moral recovery shorten the time individuals spend in the corrupt class, reduce both the peak and endemic levels of corruption, and alleviate the downstream burden on the prison system. Prosecution remains a necessary complement, but it operates after corruption has occurred, whereas reform reduces the number of individuals who become persistently corrupt. At a sufficiently high reform rate, the model predicts that active corruption can be driven toward elimination. Future work may extend the model through optimal control, stochastic parameterization, fractional-order operators, or empirical calibration against country-specific data.

Data Availability

The original contributions presented in this study are included in the article/supplementary material. Further inquiries can be directed to the corresponding author(s).

Conflicts of Interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest

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