Modeling and Analysis of the Interaction of Neutral and Protester Populations: A Competing Species Model

A. Kazmierczak
Department of Computer Science
Oklahoma State University
Stillwater, OK
akazmie@okstate.edu

Abstract

The rise of radicalized terrorist groups, such as the Islamic State of Iraq and Syria (ISIS), throughout the world have brought concern, debate, and contention to the modern world. The recruitment strategies of terrorist networks are global and are no longer concentrated in a particular location. In this paper, we present a dynamical model of the interaction and recruitment between a non-radicalized or neutral and radicalized population. The formulation is based on models of interactions between competing species [3] type dynamics. An exploration of the long-term dynamics and stability of homogeneous equilibrium solutions and their stability is given. The paper is given in two parts. Part one analyzes the current populations. Part two analyzes the situation when an additional number of radicals are introduced into the radicalized population.

Keywords: Terrorism, competing species model, equilibrium solutions, stability at equilibrium solutions.

Mathematica subject classification: 62J12, 62G99

Computing Classification: I.4

1. Introduction

Protestorsare not a new phenomena. However, there is a marked and exponential increase in the frequency of protests since the inception of the civil rights act. Protestors can wreak havoc and spread fear and panic to native citizens. In addition, the strength and presence of protestor activities create emigration issues. Consequently, countries are faced with extremely difficult, complex, and contentious political and social decisions on the activities that cause protest situations.

Despite these ongoing protests, there is not much literature that takes a dynamical systems approach to understanding the spread of terrorism, at a population. Our primary objective is to bridge the gap.

In our framework, we let N represent the neutral population. The protestor is denoted by P: P can be viewed as the protestor population of a certain situation. This paper is a first step in providing a mathematical modeling framework to study the evolution and interaction between this cop and protestor population. The protestor population is modeled by standard population growth models. Also, we consider the addition to the protestor population from increased protestors. The paper is organized as follows. In section two, we develop and analyze the time-dependent autonomous protestor ordinary differential equation (ODE) modeland the effect of a current populations. We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically. In section three, we consider the situation when more protesters are in the system.We examine the equilibrium solutions, the stability of the equilibrium solutions and investigate the dynamics numerically for this situation also. In section four, we consider the scenario when the protestor population declines. In section 5 we present ur conclusions.

2. **Neutral Protester (N, P) ODE Model**

Consider the mathematical model

$$N = a_1N/(1+d_1C) – a_{NR}NP/(1+d_2N) – b_1N^2= 0 = f_N(N, P) \quad (1)$$

$$P = a_2P/(1+d_3N) - a_{NR}NP/(1+d_2N) – b_2P^2 = 0 = f_R(N, P) \quad (2)$$

The populations N(t) and P(t) represent the populations of the neutral and protester populations.. The parameters are all assumed to be positive and their descriptions are given in Table 1a.

Table 1a: List of parameters used in the differential equation model

Symbols	Meaning
a_1	Growth rate of the protestor population
a_2	Growth rate of the police population
b_1	Population loss in N due to intra-species competition and natural mortality
b_2	Population loss in P due to intra-species competition and natural mortality
a_{NR}	Maximum per capita loss in N due to recruitment by protester groups
d_1	Measures the effectiveness of N in disrupting the growth rate of N
d_2	Measures the resilience of N to recruitment strategies by P
d_3	Measures the effectiveness of Nin disrupting protester activities

In the case of $d_i = b_i = 0$, the mathematical model becomes similar to the competing species model. The parameters d_i influence the carrying capacity of the individual populations. For instance, if d_1>> 1 then the growth rate of N is reduced. This is interpreted as: a highly effective protester population can greatly hinder the growth rate of N. The growth rate of the protester population depends on the successful recruitment from the neutral. Notice, that if d_2>> 1 then the recruitment by P is small, Also, if d_3>> 1, new protester are introduced into the protester population is smaller. The values chosen for the variables in this model are listed in Table 1b.

Table1b: Values of parameters

a_1	a_2	b_1	b_2	a_{NR}	d_1	d_2	d_3
2	2	0.5	0.5	2	2	2	3

2.1Neutral Protester (N, P) ODE Model

Consider the mathematical model

$$f_N(N. P) = (a_1/(1+d_1P) – a_{NR}P/(1+d_2N) – b_1N) N = 0 \quad (3)$$

$$f_R(N, P) = (a_2/(1+d_3N) –anrN/(1+d_PN) – b_2P) P = 0 \quad (4)$$

Since this system is nonlinear, the first step is linearization using the Jacobian.

The Jacobian for this system is defined as

$$J = \begin{vmatrix} \partial f/\partial N & \partial f/\partial P \\ \partial g/\partial N & \partial g/\partial P \end{vmatrix}$$

Taking the partial derivatives, simplifying and using the values in table for the parameters, the Jacobian becomes.

$$J = \begin{vmatrix} 2/(1+2P)-2P/(1+2N)^2-N & -2/(1+2P)^2-2N/(1+2N) \\ -6P/(1+3N)^2-2P/(1+2N)^2 & 2/(1+3N)-2N/(1+2N)-P \end{vmatrix}$$

2.2 Equilibrium Points

Using Maple CAS, on (3) and (4) weobtained the following real valued equilibrium points:

{N = 0., P = 0.},
{N = 0., P = 4.},
{N = 4., P = 0.},
{N = .4301871556, P = .8213492010},
{N = -.4311081397, P = -1.121275136},
{N = -.4346164212, P = .1299378971},
{N = -3.952306486, P = -2.658090053}

2.3Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable. Substituting equilibrium points into the Jacobian and solving for eigenvalues, we get the results in Table 2.

2.4Summarization

Table 2 summarizes the results for the current population levels.

Table 2 – Results for Current Population Levels

Equilibrium Point	Eigen Values	Node Type	Stability
{N = 0., P = 0.},	2.00, 2.00	Repelling	Unstable
{N = 0., P = 4.}	-44/9+(2/9)*sqrt(185), -44/9-(2/9)*sqrt(185)	Attracting	Asymptotically stable
{N = 4., P = 0},	-2, -86/117	Attracting	Asymptotically stable
{N = .4301871556, P = .8213492010},	.757984137794684, -1.31667584619468	Saddle	Unstable
{N = -.4311081397, P = -1.121275136},	124.789757665452, -7.28136719345222	Saddle	Unstable
{N = -.4346164212, P = .1299378971}	-6.620132656550+9.18652446854370*I, -6.620132656550-9.18652446854370*I	Attracting spiral	Asymptotically stable
{N = -3.952306486, P = -2.658090053}	3.45507685676904, 1.47441380823096	Repelling	Unstable

3. Growth of the Protester Population

In this section, we consider the situation where a there is a 25% increase in the protester population. The mathematical model now becomes

$$f_N(N, P) = (a_1/(1+d_1(1.25P)) – a_{NR}(1.25P)/(1+d_2N) – b_1N) N = 0 \quad (5)$$

$$f_R(N, P) = (a_2/(1+d_3(N) - a_{NR}N/(1+d_2N) – b_2(1/+1.25P) \ (1.25P) = 0 \quad (6)$$

Using the Maple CAS, on (5) and (6) we obtained the following real valued equilibrium points

{N = 0., P = 0.},
{N = 0., P = 3.200000000},
{N = 4., P = 0.},
{N = .3156552235, P = 1.024389733},
{N = -.4403859177, P = 1.855697361},
{N = -.4325472689, P = -.4910205568},
{N = 1.304975560, P = -.5057028313},
{N = -5.192142042, P = -1.990030373}

3.1Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable.

3.2 Summarization

Table 3 summarizes the results for an increased police population level.

Table 3 – Results for Increased ProtesterPopulation Levels

Equilibrium Point	Eigen Values	Type of Node	Stability
(N = 0., P = 0.)	2.00, 2.00	Repelling	Unstable
(N = 0., P =3.200000000)	-6.31260878122594, -1.01712094877406	Attracting	Asymptotically stable
(N = 4., P = 0.)	-2, -86/117	Attracting	Asymptotically stable
(N = .3156552235, P = 1.024389733)	-1.60703900385485, .793359729354847	Saddle	Unstable
(N = -.4403859177, P = 1.855697361)	-249.390466169796, -11.5240335472042	Attracting	Asymptotically stable
(N = -.4325472689, P = -.4910205568)	82.9709967044000+730.356706992694*I, 82.9709967044000-730.356706992694*I	Repelling	Unstable
(N = 1.304975560, P = -.5057028313)	-156.658811051449, -19.7304049866513	Attracting	Asymptotically stable
(N = -5.192142042, P = -1.990030373)	4.53052768941096, .781943173589040	Repelling	Unstable

4. Decline of the Protester Population

In this section, we consider the situation where there is a 25% in the protester population. The mathematical model now becomes

$$f_N(N, P) = (a_1/(1+d_1(0.75P)) – a_{NR}(P)/(1+d_2(N) – b_1(N) N = 0 \quad (7)$$

$$f_R(N, P) = (a_2/(1+d_3(N)) - a_{nr}N/(1+d_2 N) – b_2(0.75P)) (0.75P) = 0 \quad (8)$$

Using the Maple CAS, on (7) and (8) we obtained the following real valued equilibriumpoints:

{N = 0., P = 0.},
{N = 0., P = 5.333333333},
{N = 4., P = 0.},
{N = .4301871556, P = 1.095132268},
{N = -.4311081397, P = -1.495033515}, {
{N = -.4346164212, P = .1732505294},
{N = -3.952306486, P = -3.544120071}

4.1 Analyzing equilibrium points for stability

In this section we use the equilibrium points to generate the eigenvalues for the system and establish whether the equilibrium point is stable or unstable

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4.2 Summarization

Table 4 summarizes the results for a decreased protester population.

Table 4 _ Results for Decreased Protestor population

Equilibrium Point	Eigen Values	Type of Node	Stability
(N = 0., P = 0.),	2, 2	Repelling	Unstable
(N = 0., P = 5.333333333)	-10.5817315235956, -3.24683990940437	Attracting	Asymptotically stable
(N = 4., P = 0.)	-2, -86/117	Attracting	Asymptotically stable
(N = .4301871556, P = 1.095132268)	.561198866339937, -1.68177931653994	Saddle	Unstable
(N = -.4311081397, P = -1.495033515)	166.047756962696, -8.18559401169599	Saddle	Unstable
(N = -.4346164212, P = .1732505294),	-9.22574755770000+9.57258822194460*I, -9.22574755770000-9.57258822194460*I	Attracting	Asymptotically stable
(N = -3.952306486, P = -3.544120071)	3.45408293768524, 2.53347890031476	Repelling	Unstable

5. Conclusions

In this paper we modeled and analyzed the interaction of protestor and neutral populations. A comparison of the results in Table 2, Table 3,and Table 4 seem to indicate that no matter the relative sizes of the populations, there will besome level of instability in the system.

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