Using Energy Conservation Concept and Basic Mathematical Modeling Techniques to Study the Effect of Greenhouses Gases to Earth's Climate System.

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ABSTRACT

The work, implemented the basic mathematical modeling techniques to Physical phenomenon (Conservation of Energy) base on Stefan Boltzmann Law in to Earth's Climate System using analytical method to study the impact of climate change, effect of greenhouses gases in Earth's surface temperature, and spreading societal awareness of its dangers.

Earth's climate system is a complexity system, that is schematically made up of five components: the atmosphere; the hydrosphere (oceans, lakes, and other bodies of water); the cryosphere (snow and ice); the lithosphere (land surface); and the biosphere (all living things). These components do not exist in isolation; they are interconnected and interact at several levels, either directly or indirectly [1,16]. The system as a entire is powered by solar radiation and develops under the influence of its own internal dynamics through ocean currents and atmospheric circulation. On the other hand, there are external factors which drive the system; these are called forgings, include both natural phenomena such as cyclical changes in the Earth's orbit around the Sun, volcanic eruptions, variations in the solar output, and human-induced (anthropogenic) factors like changes in atmospheric composition, human activities, and so on. Climate changing is one of the greatest threats towards the survival of mankind on the earth, it is well understood that human beings activities hold the main responsibility behind these climate changing issues beside the natural disasters [1].

Theoretical models of the Earth's climate system most often involve systems of proportional models, ordinary differential and partial differential equations and qualitative behavior of their solutions. Mathematical modeling is one of several approaches that have been used to study the climate system and become the most important techniques for complicate problems in modern science. However, enable a physically based estimate the range of future climate change,, providing invaluable scientific information towards political and societal decision maker for well plan in order to protect our Planet.

According to the study, it has been found that, the Green house gases factor ($0 \leq \beta \leq 1$) and other relevant human activities play a prominent role in Earth's climate change. Furthermore

Mathematical calculation provided actual value for greenhouse factor $\beta = 0.76$, which lead to moderate temperature in Earth's surface all seasons.

Key Word: Energy conservation, Mathematical Modeling, Greenhouses gases, Earth's Climate system, Global warming.

1. INTRODUCTION

Today climate change became an important issue in our planet, the study economic and social effects on societies as a whole and also its effect on global peace and security. Earth's climate system is a complexity system, that is schematically made up of five components: the atmosphere; the hydrosphere (oceans, lakes, and other bodies of water); the cryosphere (snow and ice); the lithosphere (land surface); and the biosphere (all living things). These components do not exist in isolation; they are interconnected and interact at several levels, either directly or indirectly.Climate changing is one of the greatest threats towards the survival of mankind on the earth. This motivates us to investigate climatic changing effects via fluctuations of the temperature, wind patterns and precipitation taking place in a given geoenvironment [1,12].

Practically the Sun is the main resource of energy that powers the Earth's climate system. This energy comes in the form of electromagnetic radiation, which originates from different depths in the Sun's interior. According to Physicist (**Stefan Boltzmann)**, energy conservation theory, energy can change from one form to another but the total amount of energy remains constant.

A complete climate model contains physical descriptions of all five components mentioned above and takes into consideration their coupling. Some components may be described in a simplified form or even be prescribed [16].

Mathematical model is a tool to describe physical phenomena, providing the world with a clear mathematics view on the current state of knowledge in climate system, climate change, weather predictions and its potential environmental and socio-economic impacts.

2. Problem Statement:

Although much progress has been made over the decades, our Planet still faces multiple societal, economic and environmental challenges. Climate changing is one of the greatest threats towards the survival of animals and mankind on the Earth. Climate change has already started to cause a wide range of physical effects with serious implications for investors and businesses.

3. **Objective of Study:**

The study focus on well known physics phenomenon (energy conservation theory base on Stefan Boltzmann Law) implemented to temperature of Earth's surface using essential mathematical modeling techniques in order to achieve the following:

(i) Employ step by step mathematical model techniques to study the effect of greenhouse gases to Earth's climate and climate change.

(ii) Mathematically, evaluate the suitable greenhouse gases factors which moderate Earth's surface temperature.

(iii) Spread the weariness between the communities and countries about the activities that will release more greenhouse gases. Also its best practices in clean energy-related research and its subsequent contribution to the unanimous goal set in Paris Climate Change Conference 2020, of achieving net zero greenhouse gas emissions by 2050.

4. Definitions and Concepts:

4.1. **Climate**

This is a word from ancient Greek "**klima**", meaning inclination. Climate is commonly defined as the weather averaged or the statistics of weather over a long period. The standard averaging period is 30 years, but other period may be used depending on the purpose. It is measured by assessing the patterns of variation in temperature, humidity, atmospheric pressure, wind, precipitation, atmospheric particle count and other meteorological variables in a given region over long period of time. (climate definition in many references).

4.2.Global Solar Radiation balance of the climate system

Radiation is the transfer of energy by electromagnetic waves, which do not require a medium, such as air, for their transmission. Solar radiation is relatively short wavelength radiation emitted by the Sun. Thermal-infrared radiation is relatively long-wavelength radiation emitted by the Earth, atmosphere, and clouds. The Earth's surface receives solar radiation during the day only, but its surface and atmosphere emit thermal-infrared radiation during day and night see [12]. The Sun is the only relevant energy source for the climate system on a temporal scale of less than about 10^{6}years. The different energy fluxes are Coming from the Sun, on average 341 W/m2 reach the top of the atmosphere, while barely half of this is available for heating of the Earth's surface. Major parts of the short-wave radiation are reflected by clouds or reflected directly on the Earth's surface itself and are absorbed by the atmosphere. Incoming radiation contrasts with surface long-wave outgoing radiation of around 396 W/m2. Virtually Earth's climate system receives all its energy from the Sun. This energy comes in the form of electromagnetic radiation, which originates from different depths in the Sun's interior. As fact some of the energy is absorbed in the solar photosphere, further absorption in the Earth's atmosphere gives the solar spectrum at the Earth's surface its more ragged appearance, for more information see [1, 16].

The points above give us main idea about the energy resource and energy balance, the mathematical explanations is Input Energy equal to Output Energy (Energy Conservation Law), and there are many factors affect the balance.

4.3. Global warming

Global warming is the increase in the average temperature of the Earth's near-surface air and the oceans it can also be defined as a gradual increase in the overall temperature of the earth's atmosphere generally attributed to the greenhouse effect caused by increased levels of carbon dioxide CO2, chlorofluorocarbons CFCs, and other pollutants [1,13].

In our point of view, the release of greenhouse gases is main factor that causing the global warming which lead to increasing in the overall temperature of the earth's as well as climate change and its Consequently such as (desertification, drought, flood, volcanoes, Tornadoes, Forest fires).

GREENHOUSE GASES

Greenhouse gases are thought to be the main contribution to climate change (The greenhouse effect). They are very efficient in trapping heat into the atmosphere; therefore, it results in the greenhouse effect. The solar energy is absorbed by the earth's surface and then reflected back to the atmosphere as heat. Then as the heat goes out to space, greenhouse gases absorb a part of the heat. After that, they radiate the heat back to the earth's surface, to another greenhouse gas molecule, or to space (The Green Effect). **Daniela Burghila et al**. stated in "climate change effect- where to next", the biggest concern scientists have is about the emission of CO_2 since it is about 75% of the total global emission of greenhouse gases [7, 9, 10].

The major greenhouse gases in the earth's atmosphere are:

- Water vapour (H_2O)

- Carbon dioxide (CO_2)
- Methane (CH_4)
- Nitrous oxide (N_2O)
- Ozone (O_3)
- Chlorolfluorocarbons (CFCs)
- Hydrofluorocarbons (HFCs) [10].

As we mentioned that the release of greenhouse gases are very efficient in trapping heat into the atmosphere, Therefore, reducing the emission of these gases become the major goal in the study specially Carbon dioxide (CO_2).

2.5. Climate model

A climate model is essentially a representation of the many interactions and dynamics within the climate which includes the atmosphere, ocean, land surface, and ice to make predictions of possible climate change for the future. Climate models are systems of proportional equations, differential equations based on the basic law of physics, fluid motion and chemistry. Mathematician are using mathematical models to modulating the scientific Phenomena in Earth's climate system to give good insight and predictions about climate change and its consequently implication.

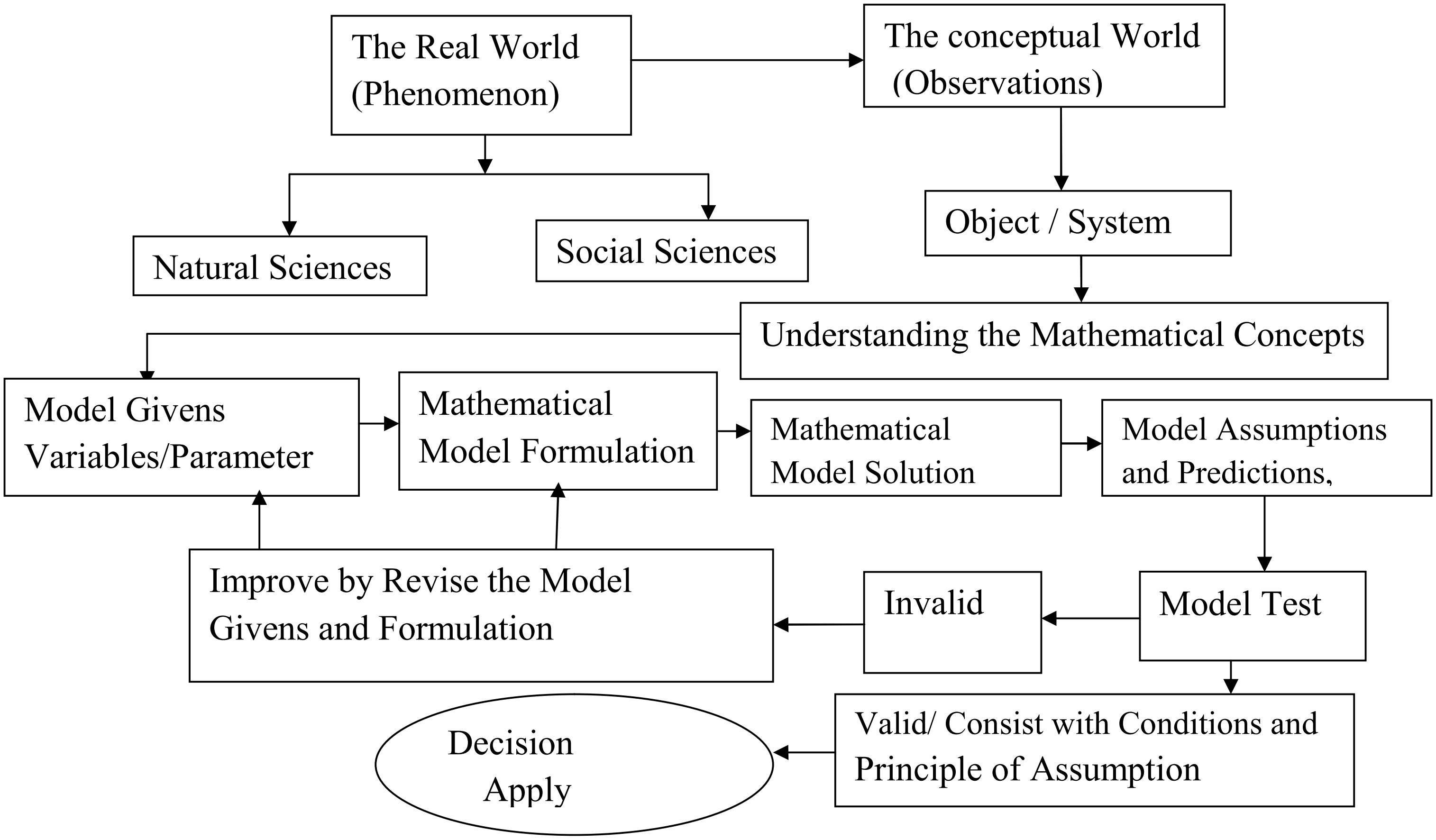
2.6. Theorem: First Law of Thermodynamics (Energy Conservation):

The first law of thermodynamics state that the energy can neither be created nor destroyed, but can be converted from one form to another. In another form, during an interaction, energy can change from one form to another but the total amount of energy remains constant [5,8].
Therefore the first thermodynamics law is obtained on the experimental basis. In the other words, we can say that the energy of an isolated system is always constant.

2.7. Mathematical Model

Models describe our beliefs about how the world functions. In mathematical modeling, we translate those beliefs into the language of mathematics [2, 3].
In the real world, these beliefs are phenomena in natural sciences or socioeconomic sciences along with some observations.

Here we designed the following diagram represents the general description of mathematical model techniques for all Phenomena in science and socioeconomics sciences step by step till test the validation of the model.



The above diagram is the complete mathematical modeling process for any phenomenon in our real world. The process starts with the real world in Natural or Social Sciences along with with a physical system and some observations or an experiment notice. When the laws of physics that are thought to govern the behavior of the system are translated in mathematical terms, the result is what is called a mathematical model. By Understanding the Mathematical Concepts deeply such as the model givens, formulation and solution of the model. The mathematical model is subsequently analyzed for its properties and used to generate predictions about the behavior of the system in a changing environment. These predictions are tested against observations, so if there is no discrepancy between predictions and observations, the model will accept; otherwise, the model will refine or improve, returning to model formulation or givens by considering all the impressive factors in our model in order to achieve good results which agreed with real observations by repeating the process until the

best result attain. Thus, mathematical modeling is an iterative process. Mathematical climate models enable a physically based estimate and predict the future climate change [4,5,6].
Therefore we applying the above diagram or flowchart to energy conservation theory in Earth's climate system in sequence to give an approximate solutions or predictions for sophisticate problem in Earth climate system, to help decision maker for well plan in order to protect our planet.

2.8 Analytical Techniques:

Energy Balance Model (Energy Conservation): In Earth's climate system, model builds a series of zero-dimensional energy balance models. The Earth's climate system is described in terms of a single variable, namely the temperature of the Earth's surface averaged over the entire globe. Conceptually the Sun emits the radiation in the ultraviolet (UV) regime (wavelength less than 0.4 μm). This energy reaches the Earth's surface, where it is converted by physical, chemical, and biological processes to radiation in the infrared (IR) regime (wavelength greater than 5 μm). This IR radiation is then reemitted into space. In equilibrium state the Earth's climate the average temperature of the Earth's surface does not change, so the amount of energy received must equal the amount of energy re-emitted see [10]. A complete climate model contains physical descriptions of all five components mentioned above and takes into consideration their coupling. Some components may be described in a simplified form or even be prescribed [16,15,14].

Mathematical Formulation: To well understand the Mathematical concept about our physical phenomena (Energy Conservation Theory) in Earth's climate system, we have to classify the model components, such as units, variables, and constants, these factors may influence direct or indirect in our model [11],

- T, the temperature of the Earth's surface averaged over the entire globe, in Kelvin (K) or Celsius scale,(variable).
- R, the radius of the Earth, parameter (constant).
- A, the energy flux density (also referred to as the energy flux)|the amount of energy (W) flowing through a flat surface of area $1m^2$. From satellite observations we know that the energy flux from the Sun is $A = 1367.6Wm^{-2}$. parameter (constant).
- σ, sigma, Stefan Boltzmann constant; its value is $5.67 \times 10^{-8}Wm^{-2}K^{-4}$parameter.
- The Earth's viewed as a disk from the Sun
- The area of the disk as seen by the Sun is πR^2.

- The amount of energy from the Sun to the Earth (disk) is the Incoming Energy (W)
  $E_{In} = \pi R^2 A$
- As fact, all bodies radiate energy in the form of electromagnetic radiation.
- The temperature of the body is essential in our study, because the amount of energy radiated deepened on it.

- In Physics, shows that black - body radiation is given by the Stefan-Boltzmann law, in Wm^{-2} units [11]. $F_{SB}(T) = \sigma T^4$.
- The area of the Earth's surface is $4\pi R^2$
- The amount of energy radiated out by the Earth surface is outgoing energy(W)
  $E_{out} = 4\pi R^2 \sigma T^4$

The Model Formulation and Solution:

The First Law of Thermodynamics speaks about the conservation of the energy: "Energy cannot be destroyed or created; it can only be transformed". At thermal equilibrium, the incoming energy (temperature) must equal to outgoing energy and with use Stefan Boltzmann Law, such that

$E_{in} = E_{oui}$

Therefore the mathematical model represent the following equation

$\pi R^2 A = 4\pi R^2 \sigma T^4 \;\Rightarrow A = 4\sigma T^4$

TO solve the equation for T, thus $T = (\frac{A}{4\sigma})^{\frac{1}{4}}$

Where $\sigma = 5.67 \times 10^{-8}$and $A = 1376.6$

The value of $T = (\frac{1376.6}{4\times 5.67\times 10^{-8}})^{\frac{1}{4}} \approx 278.7K$

Interpretation: The Earth's temperature increase if the incoming energy is greater than out coming energy and decrease if the incoming energy is lower than out coming energy. Whoever the Earth's temperature remains constant if the incoming energy balances the out coming energy and the planet said to in thermal equilibrium, this as normal understanding. The value of $T = 278.7K$ which equals to 5.5 degrees Celsius, but actual average Earth's temperature approximately 16 degrees Celsius, this is a huge difference between the predictions (the calculated value) and observation (the actual value).

Decision: According to the interpretation above, the model*1* is invalid. Therefore the result should be rejecting and the model must be improve.

Model*2*

In order to get a better model, returning to the model givens, formulation and give deep insight to the problem. In model *1* we omitted a number of important factors. These factors represent incoming energy from Sun reflected back out to space. Snow, ice, and clouds, for example, reflect a great deal of the incoming light from the Sun. We use the term albedo (" The fraction of the incoming solar energy scattered by Earth back to space is referred to as the planetary albedo"). to measure the Earth's reflectivity.

Model *2* formulation:

By adding the reflectivity factor into the constant of above model*1*

- ρ: albedo. The Earth's average *albedo* is about 0.3, which means that roughly 70% of the incoming energy is absorbed by the Earth's surface.

The Model *2* Build:

The amount of energy reaching the Earth is incoming energy (W)

$E_{in} = \pi R^2 A(1-\rho)$

The amount of energy radiated out by the Earth is Outgoing energy (W)

$E_{Out} = 4\pi R^2 \sigma T^4$

The Model Solution:

According to conservation of the energy and thermal equilibrium state, the incoming energy equal to out coming energy and Stefan Boltzmann Law such that

$$E_{In} = E_{Out} \Rightarrow \pi R^2 A(1-\rho) = 4\pi R^2 \sigma T^4 \Rightarrow A(1-\rho) = 4\sigma T^4$$

Therefore $T = \left(\frac{A(1-\rho)}{4\sigma}\right)^{\frac{1}{4}} = \left(\frac{1367.6\times 0.7}{4\times 5.67\times 10^{-8}}\right)^{\frac{1}{4}} \approx 254.9K$

Interpretation: Although we consider the *albedo* factor or the energy absorbed by the Earth's surface, but the value of temperature $T = 254.9K$, this equivalent to -18.25 degrees Celsius, its prediction of the temperature value at equilibrium is worse than the prediction of Model*1* ,the difference is still wide mathematically.

Decision: By test the model *2* , we rejecting the result and revise the model*2* in order to refine or improve the predictions.

Model *3*: The result in model 2 did not give the expected accuracy value, so it's better to look carefully to another factors The only option is to look where we might have overlooked something in the model. In this cycle, we focus on the outgoing radiation, which come from chemical reaction, nature and human activities. .

Mathematical Formulation:

Greenhouse gases come from chemical reactions, nature such as Volcanoes or human activities such as burning fossil fuels, like carbon dioxide (CO_2,H_2O, CH_4, NO_2) methane, and water, as well as dust and aerosols have a significant effect on the properties of the atmosphere.

Model*3* build and solution:

Experimentally β ,Greenhouses factor ranging $0 \le \beta \le 1$

Incoming energy (W): $E_{In} = \pi R^2 A(1-\rho)$

The amount of energy radiated out by the Earth is Outgoing energy (W):

$$E_{out} = 4\pi R^2 \beta \sigma T^4$$

Model*3*Solution:

According to conservation of the energy and thermal equilibrium state, the incoming energy equal to out coming energy and Stefan Boltzmann Law, such that

$$E_{In} = E_{out},$$

$$\text{Thus } T = \left(\frac{A(1-\rho)}{4\beta\sigma}\right)^{\frac{1}{4}} \approx 282.9K$$

Interpretation:

The value of $\beta = 0.66$ gives a climate model that correctly predicts the current global average temperature $T \approx 282.9K$, which equivalent to 9.64 degrees Celsius, this gives reasonable approximate value to the Earth's surface temperature. Mathematical calculation provided actual result for greenhouse factor $\beta = 0.76$, this may lead to moderate temperature in Earth's surface all seasons.

The effect on the outgoing radiation is difficult to model because it depend on the Greenhouse gases mostly depend on human activities which are increasing daily. Meanwhile, it's manageable, if there is a strong determination to reduce this factor to reasonable value, since its human factor,. However many conferences recommended, reducing the greenhouse gases which mainly belong to human activities.

Result Discussion: Step by step mathematical model techniques has been implemented Overall result in model1*1*, model*2* and model *3*, it's clearly indicate the effect of greenhouse gases factor β which range $\leq \beta \leq 1$ very huge. However, this factor is controllable or reducible to appropriate value because mainly depend on human activities.

Conclusion: Concept of energy conservation, Stefan Boltzmann Law, and step by step mathematical modeling techniques have been implemented, concentrated on the inconsistency of the results between the prediction and observations. On the other hand the model has been improved by considering the other factors, which can affect the results.

The study shows, the effect of greenhouse gases to the Earth's climate system, by increasing Earth's surface temperature, which causes many problem, such as desertification, drought, flood, volcanoes, Tornadoes, and Forest fires. Mathematical calculation provide an appropriate value to greenhouses factor to be $\beta = 0.76$ in order to get the Earth's surface temperature 16 degrees Celsius.

The impacts of climate change are expected to grow more severe over the coming years and decades unless the industrial countries take the initiative seriously to reduce the greenhouses gases and spreading the wariness between the societies and communities about the advantage and disadvantage of climate change and it's consequently.

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