IJO - INTERNATIONAL JOURNAL OF MATHEMATICS
(ISSN: 2992-4421) Dhananjaiah D. S[1]*
https://ijojournals.com/ Volume 07 Issue 03 || March, 2024 ||
A Mathematical analysis of oscillatory free convection in a vertical wavy channel with Chemical reaction

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Abstract

The effect of chemical reaction on unsteady combined heat and mass transfer flow of a viscous electrically conducting fluid in a vertical wavy channel with oscillatory flux. The non-linear governing equations are solved by employing a regular perturbation technique with the slope δ of the wavy wall as a perturbation parameter. The velocity, the temperature and the concentration are analyzed for different variations of the governing parameters. The rate heat and mass transfer are evaluated for different variations.

Keywords: Heat Transfer, Mass Transfer, Chemical reaction, Wavy channel,

1. Introduction

Due to growing significances, the application of non-Newtonian liquid is mandatory in the engineering and industry. It is outstanding to those plentiful applications in more than a few regions, they are, the plastic manufacturing, performance of lubricant, food processing, and/ or movement of biological liquids. The second graded fluid preserve many fluids these are diluted polymer solution, slurry flow, as well as industrial oil, in addition to a lot of flow problems by a choice of geometry as well as dissimilar mechanical and/or thermal boundary cir- cumstances have been deliberated. Tan and Masuoka [1] found the Stokes first problems for the second graded fluids and Rashidi et al. [2] discussed by the unsteady compressible flows of the second order fluids. Hayat et al. [3] explored by the unsteady stagnation point flow of second grade fluids with changeable free stream.Due to complicated relation between stress and strain in non-Newtonian fluids and their technological application, their study in fluid dynamics is more valuable than Newtonian fluids. Viscous fluids flow has attracted the attention of scientists and engineers because of its important applications notably in the flow of oil through porous rocks, the extraction of energy from geothermal regions, the filtration of solids from liquids and drug penetration through human skin. Second grade fluid is a subclass of non-Newtonian fluid in which velocity field has up to two derivatives in stress strain tensor relationship where as in New-tonian fluid it has derivatives up to first order. Flow of second grade fluid gains attention of the researchers in many boundary layer flows and have been successfully studied in various kinds of flows. Study of heat transfer in non-Newtonian fluids is much interesting for researchers now-a-days.

The combined heat and mass transport problems through the chemical reaction are of significance in a lot of processes and have obtained an extensive value of concentration in current years. In developments such as drying, disappearance at the external of a fluid body, energy transportation in a drenched cooling increase and the flow in a desert cooler, heat and mass transport happen simultaneously. Possible applications of that category of flow can be established in numerous industries. Some examples, in the power industries, between the techniques of generating electric energy is solitary in this electrical energy are extracted directly exciting from a conducting fluid. It is predominantly attracted in cases of diffusion and chemical reaction occurs at approximately the identical speediness. Once diffusion is to a great extent faster than chemical reaction, then merely chemical reaction influences the rate of chemical reaction; when diffusion is not much quicker than chemical reaction, the diffusion as well as kinetics interacts to construct very dissimilar consequences. The investigation of heat generation or absorption consequences in moving fluids is significant in sight of quite a few substantial problems, they are, and flu- ids undergo exothermic or else endothermic chemical reaction. Outstanding to the quick development of electronic technology, effectual freezing of electronic apparatus has become certified and freezing of electronic apparatus ranges from own transistors to foremost structure computers and from energy providers to telephone switch panels and thermal diffusion impacts has been exploited for isotopes separation in the combination among gases with extremely low molecular weight (H_2 and H_e) and average molecular weight.
Bestman [4] investigated the free convection boundary layer flow with simultaneous heat and mass transfer in a porous medium when the boundary walls move in its own plane with suction. Abdus Sattar and Hamid Kalim [5] studied the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Makinde [6] explored the combined free convection boundary layer flow with thermal radiation and mass transfer past a permeable vertical plate. Makinde et al. [7] investigated the problem of unsteady convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture in an optically thin environment. Muthu- cumaraswamy and Ganesan [8] explored the impact of the chem- ical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das et al[9] have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy[10] has studied the effects of reaction on a long surface with suction. Radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

Kandaswamy et al[11] have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

The study of heat transfers and mixed convection flow in enclosures of various shapes has received attention [12] due to its practical applications. Interest in these convection flow and heat transfer in porous medium has been motivated by a broad range of applications to geothermal systems, crude oil production, storage of nuclear waste materials, ground water pollution, fiber and granular insulations solidification of castings. In a wide range of such problems, the physical

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system can be modeled as a two-dimensional rectangular enclosure with vertical walls held at different temperatures and the connecting adiabatic horizontal walls. Convective heat transfers in a rectangular porous duct whose vertical walls are maintained at two different temperatures and horizontal walls insulated received attention by several investigators [13]. Furthermore, in references [14 and 15] numerical results are being presented.

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[16],Lai and Kulacki[17].The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[18]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably, Nelson and Wood [19].

Theoretical and experimental investigations of natural convection MHD flow over a vertical porous plate in presence of chemical reaction plays an important role in Agriculture, geophysics and astrophysics. To study the underground water resources, filtration and water purification process in chemical engineering one must know the concepts of the fluid flow through porous medium. The porous medium is like a non homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid. Oscillatory flows are associated with high rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems such as pulse combustors and reciprocating engines. Many investigators reported oscillatory flows by involving various physical situations.

2.Mathematical model

We consider the motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at y =L while the boundary at y = -L is maintained at constant temperature T_1 while both the walls are maintained at uniform concentration. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. We choose a rectangular Cartesian system 0 (x, y) with x-axis in the vertical direction and y-axis normal to the walls. The walls of the channel are at y=±L.

The equations governing the unsteady flow, heat and mass transfer in terms of stream function ψ.

$$[(\nabla^2\psi)_t+\psi_x(\nabla^2\psi)_y-\psi_y(\nabla^2\psi)_x]=\nu\nabla^4\psi-\beta g(T-T_0)_y- -\beta^* g(C-C_0)_y-(\frac{\nu}{k})\nabla^2\psi \tag{2.1}$$

$$\rho_e C_p(\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\lambda\nabla^2\theta-Q(T-T_o)+Q_1(C-C_o) \tag{2.2}$$

$$(\frac{\partial\phi}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y})=D\nabla^2\phi-k_1(C-C_o) \tag{2.3}$$

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The boundary conditions for the velocity and temperature fields are

$$\frac{\partial\psi}{\partial y}=0,\quad \frac{\partial\psi}{\partial x}=0,\quad T=T_1,\quad C=C_1 \qquad \text{on } y=-L$$

$$\frac{\partial\psi}{\partial y}=0,\quad \frac{\partial\psi}{\partial x}=0,\quad T=T_2+\Delta T_e\ Sin(mx+nt),\quad C=C_2 \qquad \text{on } y=L \tag{2.4}$$

Introducing the non-dimensional variables as

$$x'=mx,\ y'=y/L, t'=t\nu m^2, \Psi'=\Psi/\nu, \theta=\frac{T-T_2}{T_1-T_2}, \phi=\frac{C-C_2}{C_1-c_2} \tag{2.5}$$

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R(\delta(\nabla_1^2\psi)_t+\frac{\partial(\psi,\nabla_1^2\psi)}{\partial(x,y)})=\nabla_1^4\psi+(\frac{G}{R})(\theta_y+N\phi_y)-D^{-1}\nabla_1^2\psi-M^2\frac{\partial^2\psi}{\partial y^2} \tag{2.6}$$

$$\delta P(\delta\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y})=\nabla_1^2\theta-\alpha\theta+Q_2\phi \tag{2.7}$$

$$\delta Sc(\delta\frac{\partial\phi}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y})=\nabla_1^2\phi-\gamma\phi \tag{2.8}$$

where

$R=\frac{UL}{\nu}$ (Reynolds number)

$G=\frac{\beta g\Delta T_e L^3}{\nu^2}$ (Grashof number)

$P=\frac{\mu c_p}{k_1}$ (Prandtl number),

$D^{-1}=\frac{L^2}{k}$ (Darcy parameter),

$Sc=\frac{\nu}{D_1}$ (Schmidt number)

$M^2=\frac{\sigma\mu_e^2H_o^2L^2}{\nu^2}$ (Hartmann Number)

$\alpha=\frac{QL^2}{\lambda}$ (Heat source parameter)

(Radiation absorption parameter)

$\gamma_1=\frac{K_1L^2}{D_1}$ (Chemical reaction parameter) $\delta=m\,L$ (Aspect ratio)

$\gamma=\frac{n}{\nu m^2}$ (non-dimensional thermal wave velocity)

$\nabla_1^2=\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ The corresponding boundary conditions are

$$\psi(+1)-\psi(-1)=-1$$

$$\frac{\partial\psi}{\partial x}=0,\quad \frac{\partial\psi}{\partial y}=0 \quad at\ y=\pm1 \tag{2.9}$$

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$$\theta(x,y)=1 \quad ,C=1 \quad on\ y=-1$$
$$\theta(x,y)=Sin(x+\gamma t) \quad ,C=0 \quad on \quad y=+1 \qquad \theta(x,y)=Sin(x+\gamma t),C=1$$
$$\frac{\partial\theta}{\partial y}=0,\frac{\partial C}{\partial y}=0 \quad at \quad y=0 \qquad (2.10)$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis. Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t.

3. Nusselt number and Sherwood number

Knowing the temperature & concentration the local rate of heat and mass transfer on the walls have been calculated using the formula

$$Nu=\frac{1}{\theta_m-\theta_w}\left(\frac{\partial\theta}{\partial y}\right)_{y=\pm1}$$

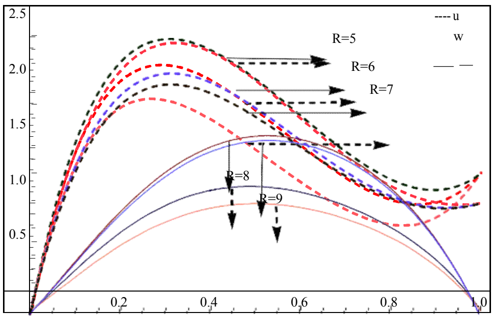
where $\theta_m=0.5\int_{-1}^{1}\theta\,dy$ and $Sh=\frac{1}{C_m-C_w}\left(\frac{\partial C}{\partial y}\right)_{y=\pm1}$

where $C_m=0.5\int_{-1}^{1}C\,dy$

where $d_1.d_2,........................,d_{14}$ are constants.

4. Discussion of the numerical results

In this analysis we investigate the effect of Chemical reaction on convective Heat and mass transfer flow of a viscous fluid in a vertical wavy channel.



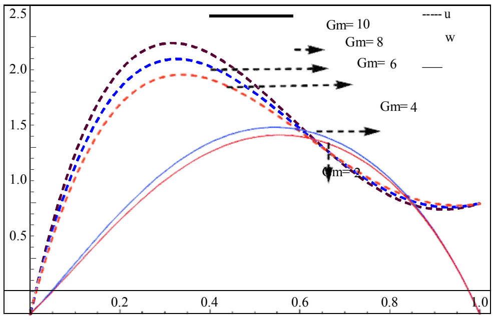


Fig.1 Velocity Profile for various values of Chemical Reaction **Fig.2 Velocity Profile for various values of Grashof Number**

increase of k^2 (rotation parameter) increase the Primary velocity but the reverse process exists for the secondary velocity. At the same time in certain stage after that reverse processes exists for the secondary velocity in the fluid flow.The increasing of permeability parameter K., Grashof number for heat transfer Gr(Figs.1&2)

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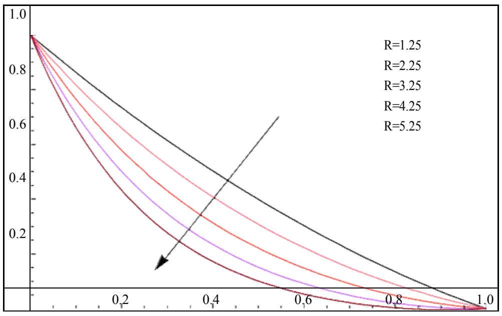

Fig.3 Temperature Profile for various values of Chemical Reaction

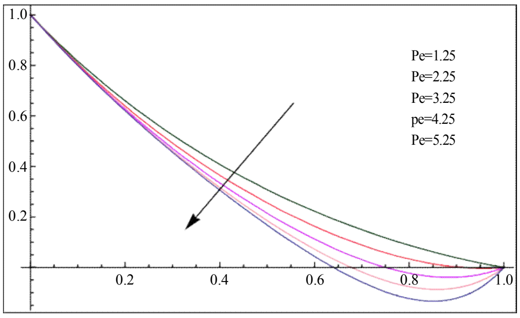

Fig.4 Concentration Profile for various values of Peclet Number

Fig.3 shows that,The increase effects of temperature exists, the reverseprocesses exists if increase of chemical reaction parameter.Concentration profile shows the decrease effects while increasing of Peclet number(Fig.4).

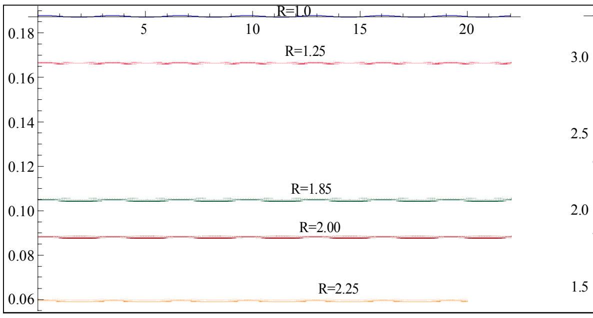

Fig.5 Mass flux for various value of Chemical Reaction

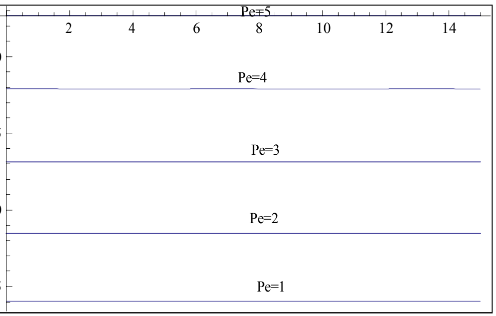

Fig.6 Heat flux for various value of Peclet Number

Mass flux shows decrease effects while increasing of chemical reaction (Fig.5). Heat flux shows increasing effects while increase of Peclet Number(Fig.6).

The Tables 1–3 symbolize the skin friction, Nusselt number and Sherwood number for dissimilar deviations in the per- tinent parameters. When the magnetic field is large, then the Hall current will be developed in the flow field.

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Table.1 The shear stresses

M	k	R	Pr	Gr	Gm	Sc	Kc	H	Q1	be	bi	S
2	1	1	0.7	5	3	0.2	1	1	1	1	0	2.532394
3												1.884042
4												1.618128
	1											2.913229
	2											3.049695
		2										2.464944
		3										2.451027
			3									1.313291
			7									1.280852
				10								4.461883
				15								6.441776
					6							3.706505
					9							4.90861
						0.3						3.081731
						0.6						13.56434
							2					3.501852
							3					5.160271
								2				1.271224
								3				1.219895
									2			6.037402
									3			9.631637
										2		2.960489
										3		3.181527
											0	2.57465
											1	2.624382

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Table.2 The Nusselt number

Kc	Q_l	Sc	H	Pr	n	t	Nu
1	1	0.22	1	0.71	0.5	0.5	0.727227
2							0.850651
3							0.924433
	2						-0.14204
	3						-0.0117
		0.3					0.813039
		0.6					1.004183
			2				1.249786
			3				1.60923
				3			3.39786
				7			7.54639
					1		0.72799
					1.5		0.728996
						1	0.727683
						1.5	0.72827

Table.3. The Sherwood number

Kc	Sc	N	t	Sh
1	0.22	0.5	0.5	0.715581
2				0.90641
3				1.054072
	0.3			0.841856
	0.6			1.25509
		1		0.715965
		1.5		0.716476
			1	0.715813
			1.5	0.716109

The skin friction magnitudes are described in Table 1. An Increases in the Hartmann number precede decreases in skin friction. because the frictional drag was decreased by the Lorentz effect on a viscous fluid. An increase in the rotation parameter, Prandtl number, and heat source parameter is used to examine the comparable behavior. Furthermore, an increase in the permeability parameter K leads to increased skin friction in significant ways on the surface boundary. Similarly, increases in the radiation-absorption parameter, Schmidt number, chemical reaction parameter, thermal Grashof number, mass Grashof number, Hall, and ion slip parameters

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near the surface boundary are examined for the same effect. Table 2 indicates that an increase in the chemical reaction parameter, Schmidt number, Prandtl number, heat source parameter, oscillation frequency, and time all contribute to an increase in the Nusselt number. It decreases as the radiation-absorption parameter increases. According to Table 3, a stronger Sherwood number is preceded by an increase in the Schmidt number, chemical reaction parameter, oscillation frequency, or time.

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