Analytical study of incompressible MHD non-Newtonian fluid in cylindrical pipe with isothermal wall and temperature-dependent viscosity

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Abstract:

In this work, analytical study of incompressible MHD non-Newtonian fluid in cylindrical pipe with isothermal wall and temperature-dependent viscosity was examined. The coupled nonlinear momentum and energy equations were solved using the traditional regular perturbation technique. Vogel's model viscosity was introduced to account for the temperature-dependent viscosity, while the third grade fluid was accommodated to model the non-Newtonian fluid feature. It was observed that the third grade and the magnetic field parameters reduced the velocity profiles and increased the temperature profiles when increased at a steady rate within the constant viscosity index but increased the velocity and the temperature profiles when subjected to the Vogel model. Meanwhile the Eckert parameter was observed to enhance the temperature near the walls of cylindrical pipe.

Keywords: Viscosity, Non-Newtonian, MHD, Isothermal, Incompressible.

1. Introduction

Flow of an incompressible MHD non-Newtonian fluid in cylindrical pipe finds application in polymer industry, petroleum industries and other types of pulp industries. In recent years, the non-Newtonian fluids have become very much important. However with its complexity, it is difficult to suggest a single model which will exhibit all the properties of non-Newtonian fluids, as such various empirical and semi empirical models have been put forward. Meanwhile, for lubricating fluids, heat generated by internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. Non-Newtonian fluid can be classified mainly into two groups such as differential type fluids and rate type fluids. Many researchers have done some work in this area (such as Fosdick and Rajagopal [5]), who examined the thermodynamics and stability of fluids of third grade. They showed restrictions on the stress constitutive equation. They were concerned with the relation between thermodynamics and stability for a class of non-Newtonian incompressible fluids of the differential type. They gave detailed attention to the special case of fluids of grade 3 and arrived at fundamental inequalities which restrict its temperature dependent. They discovered that these inequalities requires that a body of such fluid be stable in the sense that its total kinetic energy must tend to zero in time, no matter what its previous mechanical and thermal fields, provided it is both mechanically isolated and immersed in a thermally passive environment at constant temperature from some finite time onward. Massoudi and Christie [7] dealt with the effcts of variable viscosity and viscous dissipation on the flow of third grade fluid. The boundary layer equations of third grade fluid was treated by Pakdemirli [14].

Bejan [4] studied entropy generation in fundamentally convective heat transfer. Johnson *et al* [6] investigated a fluid flow which was infused with solid particles in a pipe, while approximate analytical solutions for flow of third grade fluid was examined by Yurusoy and Pakdemirli [15]. Okedayo *et al* [12] studied the effects of viscous dissipation, constant wall temperature and a periodic field on unsteady flow through a horrizontal channel. Okedayo *et al* [13] analyzed the magnetohydrdynamic (MHD) flow and heat transfer in cylindrical pipe filled with porous media. They applied the Galerkin weighted residual method for the solution of momentum equation and semi- implicit finite differece method for the energy equation. They found that an increase in Darcy number leads to an increase in the velocity profiles, while increase in Brinkman number enhances the temperature of the system. Nargis and Mahmood [8] studied the influence of slip condition on the thin film flow of third order fluid.

[9] on approximate analytical solution of natural convection flow of non-Newtonian fluid through parallel plates, solved the coupled momentum and energy equations using the regular perturbation methd. He treated cases of constant and temperature-dependent viscosities in which Reynold's and Vogel's models were considered to account for the temperature-dependent viscosity case, while third grade fluid was introduced to account for the non-Newttonian effects. Obi [10] numerically analyzed the reactive third grade fluid in cylindrical pipe. He observed that the non-Newtonian parameters considered in the analysis: third grade parameter (β), magnetic field parameter (M), Eckert number (E_c) and the Brinkman number (B_r) had psitive effects on the velocity and temperature profiles. Aksoy and Pakdemirli [1] examined the flow of a non-Newtonian fluid through a

porous medium between two parallel plates. They involed Reynold’s and Vogel’s models viscosity and derived the criteria for validity for the approximate solution.

Obi *et al* [11] on semi-analytical solution of natural convection flow of non-newtonian fluid with temperature-dependent viscosity in pipe. They solved the nonlinear momentum and energy equations using perturbation technique. They analyzed various thermo-solutal parameters involved in the dimensionless equations. Results within the constant viscosity show that increase in these parameters increases the velocity of the fluid flow as well as the temperature of the cylindrical pipe. It is observed that increase in the Reynold’s viscosity indices increases the temperature of the cylindrical pipe greatly.

2. Mathematical Formulation

Considering Aiyesimi *et al* [2], the steady flow of an incompressible MHD third grade fluid flow in a cylindrical pipe and neglecting the reacting viscous fluid assumption, the governing momentum and energy equations with the necessary boundary conditions can be represented by,

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{\beta}{r}\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)^{3}\right) - \sigma B_0^2 u = -\frac{dp}{dz} \qquad (1)$$

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \left(\frac{du}{dr}\right)^{2}\left((\mu+\beta_3)\left(\frac{du}{dr}\right)^{2}\right) - \sigma B_0 u^2 = 0 \qquad (2)$$

$$\frac{du}{dr}(0) = \frac{dT}{dr}(0) = 0;\ u(a) = 0,\ T(a) = 0 \qquad (3)$$

where, u is the velocity of the fluid; T is the temperature of the cylindrical pipe; T_0 is the Plate temperature; B_0 is the magnetic field; μ is the coefficient of dynamic viscosity; P is the pressure; and β is the material coefficient relating to third grade fluid.

The following non-dimensional variables are introduced for non-dimensionalization:

$$r = \frac{\bar{r}}{d},\ \theta = \frac{T}{T_0},\ u = \frac{\bar{u}}{u_0},\ \mu = \frac{\bar{\mu}}{\mu_0} \qquad (4)$$

Substituting (4) into (1) to (3), yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{\beta}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)^{3} - Mu = -1 \qquad (5)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + E_c\left(\frac{du}{dr}\right)^{2} + \beta B_r\left(\frac{du}{dr}\right)^{4} + Mu^2 = 0 \qquad (6)$$

$$\frac{du}{dr}(0)=\frac{d\theta}{dr}(0)=0;\; u(0)=0,\; \theta(1)=0 \tag{7}$$

3. Method of Solution

The semi-analytical solutions for velocity and temperature profiles can be of the form:

$$u(r)=u_0(r)+\beta u_1(r)+O\left(\beta^2\right),\; \theta(r)=\theta_0(r)+\beta\theta_1(r)+O\left(\beta^2\right),\; \mathrm{M}=\beta\mathrm{M} \tag{8}$$

3.1. Constant Viscosity

Substituting (8) into (5) and (6) and separating each order of β, yields

$$\beta^0: \frac{1}{r}\frac{d}{dr}\left(r\frac{du_0}{dr}\right)=-1 \tag{9}$$

$$\beta: \frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{dr}\right)+r^2\left(\frac{du_0}{dr}\right)^3-Mu_0=0 \tag{10}$$

$$\beta^0: \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right)+E_c\left(\frac{du_0}{dr}\right)^2=0 \tag{11}$$

$$\beta: \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right)+2E_c\frac{du_0}{dr}\frac{du_1}{dr}+2B_r\left(\frac{du_0}{dr}\right)^4+E_cMu_0^2=0 \tag{12}$$

Solving (9) - (12) with the condition in (7), will give;

$$u(r)=\frac{1}{4}-\frac{1}{4}r^2+\beta\left(\frac{1}{392}r^2+M\left(\frac{1}{16}r^2-\frac{1}{48}r^4\right)-\frac{1}{392}+\frac{1}{24}M\right) \tag{13}$$

$$\begin{aligned}\theta(r)=E_c\left(\frac{1}{16}-\frac{1}{16}r^4\right)+\beta\Bigg(&-E_c\left(-\frac{1}{4536}r^4-M\left(\frac{1}{128}r^4-\frac{1}{432}r^6\right)\right)-\frac{1}{128}r^6B_r\\&-E_cM\left(\frac{1}{16}r^2-\frac{1}{64}r^4\right)+E_c\left(-\frac{1}{4536}-\frac{19}{3456}M\right)+\frac{1}{288}B_r+\frac{3}{64}E_cM\Bigg)\end{aligned} \tag{14}$$

3.2. Vogel's Model Viscosity

In this section, Vogel's model is used to represent temperature–dependent viscosity, and the Massaudi and Chritie (1995) approach was applied. The equations for momentum and energy for this model are:

$$\frac{d\mu}{dr}\frac{du}{dr}\frac{1}{r}\frac{d}{dr}\left(r\mu\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)^3-Mu=-1 \tag{15}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\mu\frac{d\theta}{dr}\right) + E_c\left(\frac{du}{dr}\right)^2 + 2\beta B_r\left(\frac{du}{dr}\right)^4 + Mu^2 = 0 \tag{16}$$

$$\mu = \mu\exp\left(\frac{Q}{A+\theta} - \theta_w\right) \tag{17}$$

The Taylor series expansion of (17) yields,

$$\mu = \alpha\left(1 - \frac{Q\theta}{A^2}\right) \tag{18}$$

where,

$$\alpha = \mu\left(\exp\frac{Q}{A} - \theta_w\right) \tag{19}$$

Q and A being parameters relating to Vogel's model

$$\frac{d\mu}{dr} = \alpha\frac{Q}{A^2}\frac{d\theta}{dr};\ Q = \beta q \tag{20}$$

Substituting (8), (18) and (20) into (15) and (16), will yield,

$$\beta^0 : \frac{1}{r}\frac{d}{dr}\left(\alpha r\frac{du_0}{dr}\right) = -1 \tag{21}$$

$$\beta : \frac{1}{r}\frac{d}{dr}\left(\alpha r\frac{du_1}{dr} - \alpha r\frac{q\theta}{A^2}\frac{du_0}{dr}\right) + \alpha\frac{q}{A^2}\left(\frac{d\theta_0}{dr}\frac{du_0}{dr}\right) + \frac{1}{r}\frac{d}{dr}\left(r^3\left(\frac{du_0}{dr}\right)^3\right) - Mu_0 = 0 \tag{22}$$

$$\beta^0 : \frac{1}{r}\frac{d}{dr}\left(\alpha r\frac{d\theta_0}{dr}\right) + E_c\left(\frac{du_0}{dr}\right)^2 = 0 \tag{23}$$

$$\beta : \frac{1}{r}\frac{d}{dr}\left(\alpha r\frac{d\theta_1}{dr}\right) - \frac{1}{r}\frac{d}{dr}\left(\alpha r\frac{q\theta_0}{A^2}\frac{d\theta_0}{dr}\right) + E_c\frac{du_0}{dr}\frac{du_1}{dr} + 2B_r\left(\frac{du_0}{dr}\right)^4 + Mu_0^2 = 0 \tag{24}$$

Solving the second-order nonlinear ordinary differential Equations (21) to (24) with the condition in (7) yields;

$$u(r) = \frac{1}{4\alpha} - \frac{1}{4\alpha}r^2 + \beta\left(-\frac{q}{A^2}\left(-\frac{1}{256\alpha^4}r^2E_c + \frac{1}{768\alpha^4}r^6E_c\right) - \frac{q}{A^2}\left(\frac{1}{1152\alpha^4}r^6E_c\right)\right.$$
$$\left. + \frac{1}{24\alpha^4}r^3 + M\left(\frac{1}{16\alpha^2}r^2 - \frac{1}{64\alpha^3}r^4\right) - \frac{q}{A^2}\left(\frac{1}{576\alpha^4}E_c\right) - \frac{1}{24\alpha^4} - M\left(\frac{1}{16\alpha^2} - \frac{1}{64\alpha^3}\right)\right) \tag{25}$$

$$\theta(r) = \frac{1}{64a^3}E_c\left(1 - r^4\right) + \beta\left(\frac{qE_c^2}{A^2}\left(-\frac{1}{4096a^6}r^4 + \frac{1}{8192a^6}r^8\right)\right.$$
$$-\frac{qE_c^2}{A^2}\left(-\frac{1}{4096a^6}r^4 + \frac{1}{16384a^6}r^8\right) + \frac{1}{9a^2}r^3B_r - M\left(\frac{1}{16a^2}r^2 - \frac{1}{64a^2}r^4\right)$$
$$\left.-\frac{2051}{33570816a^6}\frac{qE_c^2}{A^2} - \frac{1}{9a^2}B_r + \frac{3}{64a^2}M\right) \quad (26)$$

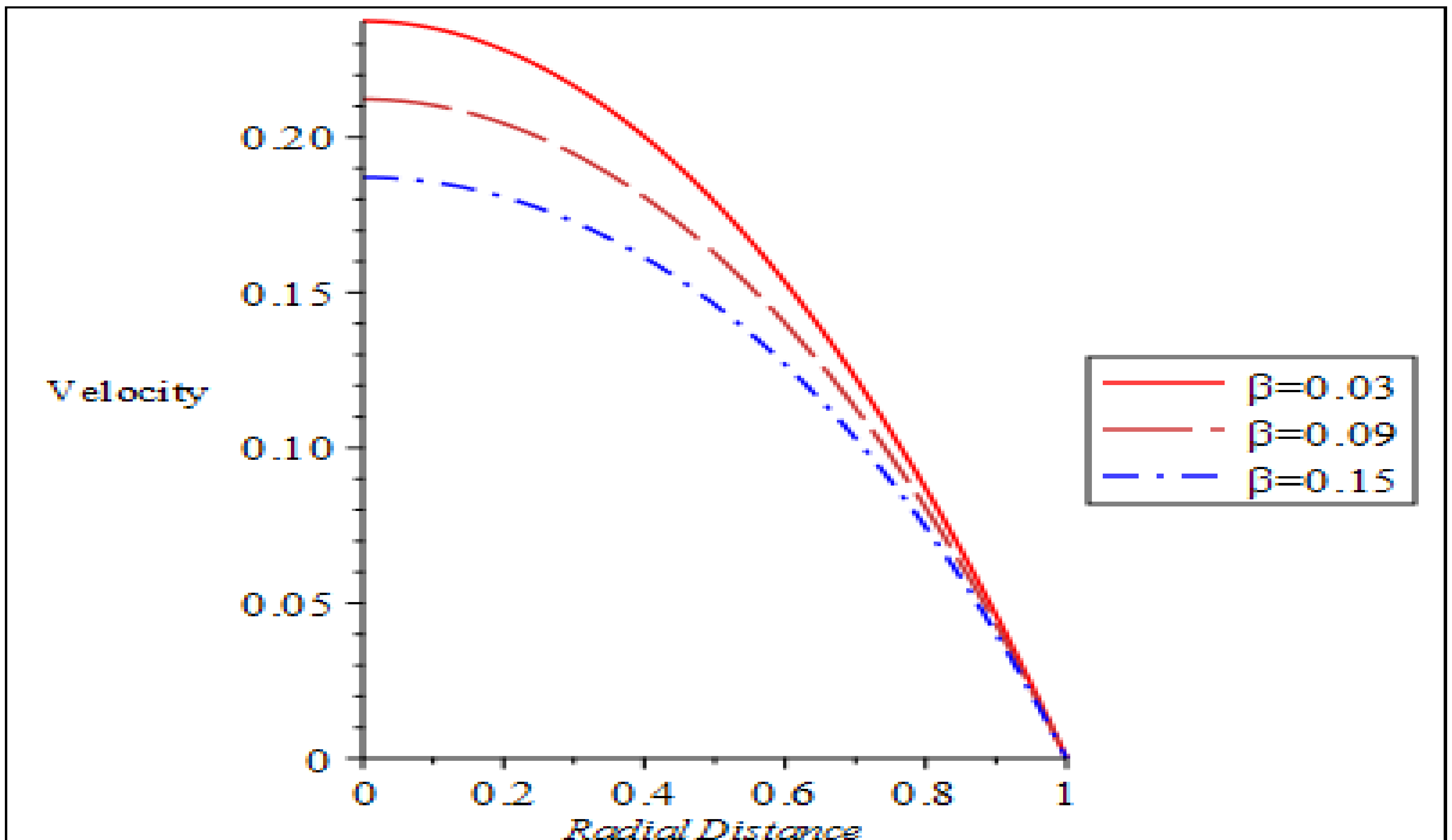

Figure 1: Velocity Profiles For Various Values of The Third Grade Parameter(β)

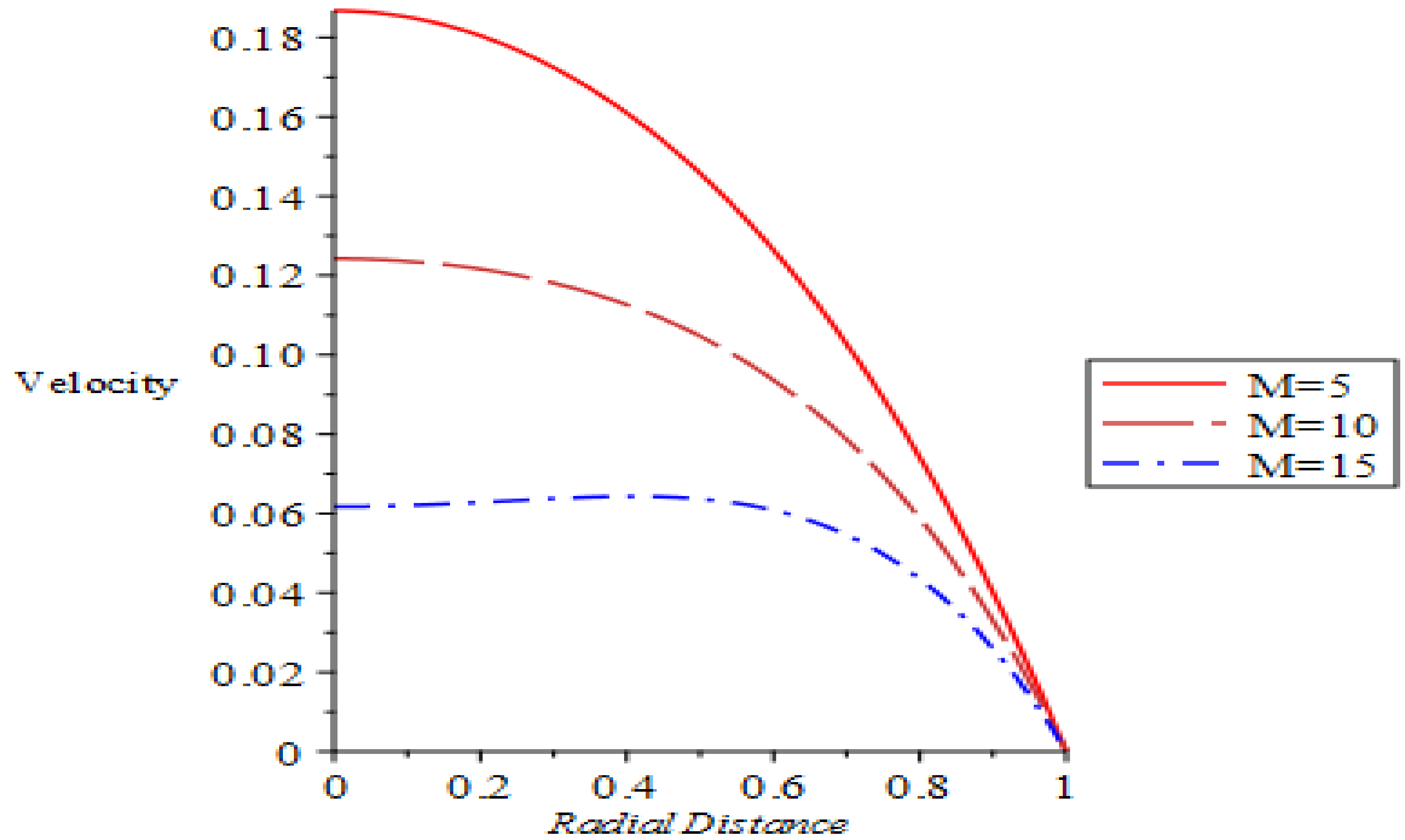

Figure 2: Velocity Profiles For Various Values of Magnetic Field Parameter(M)

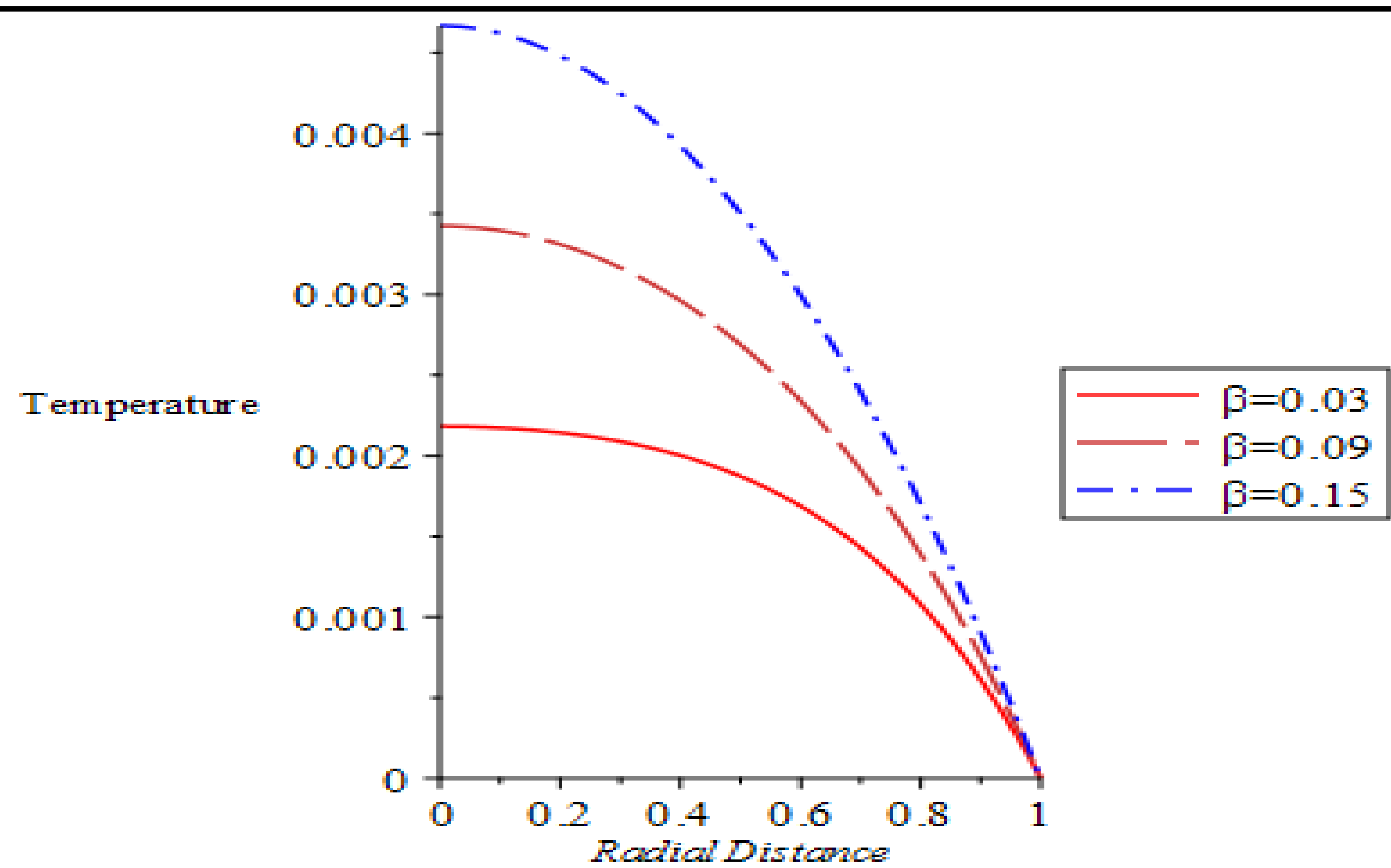

Figure 3: Temperature Profiles For Various Values of Non-Newtonian Parameter(β)

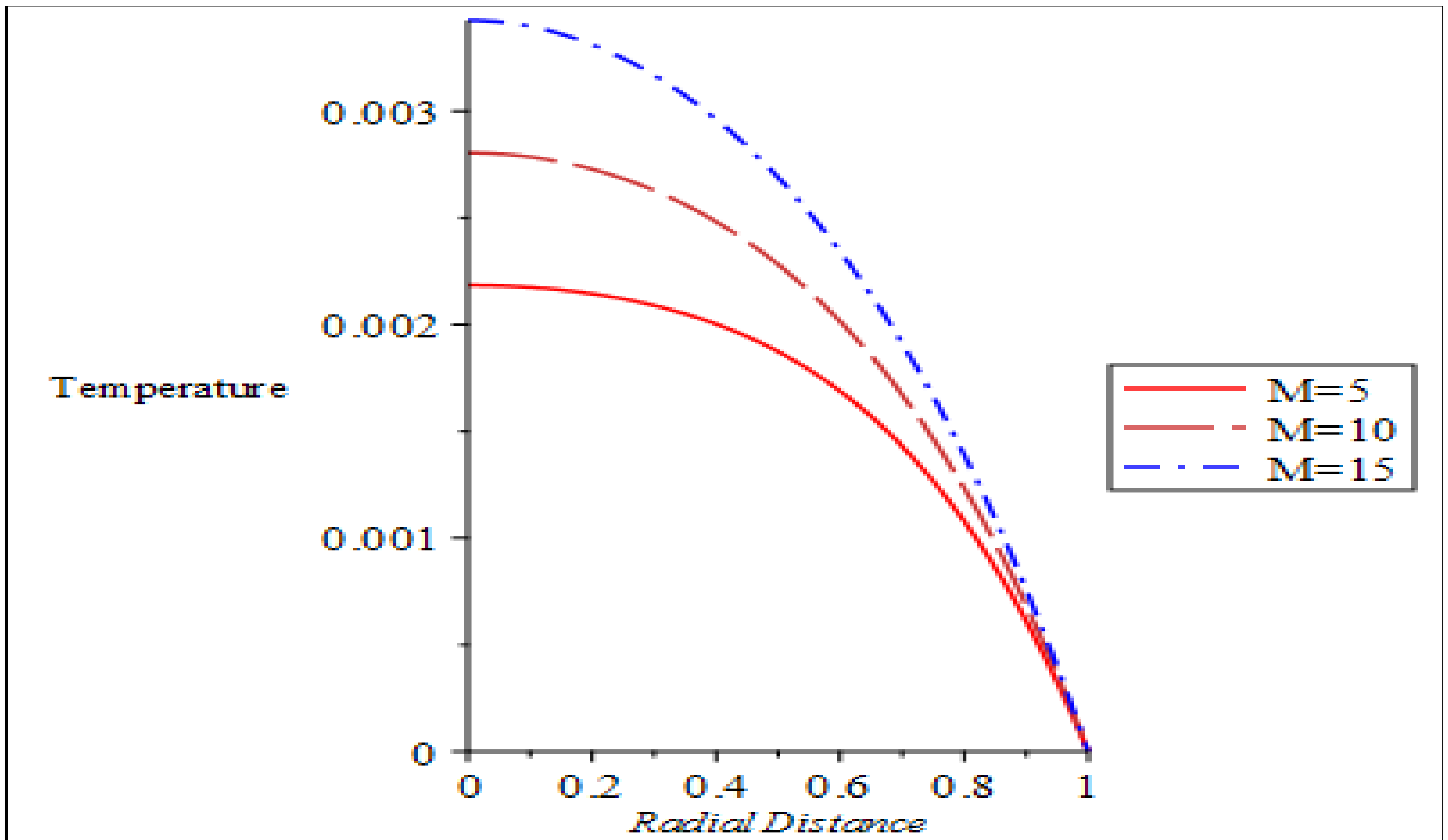

Figure 4: Temperature Profiles For Various Values of Magnetic Field Parameter(M)

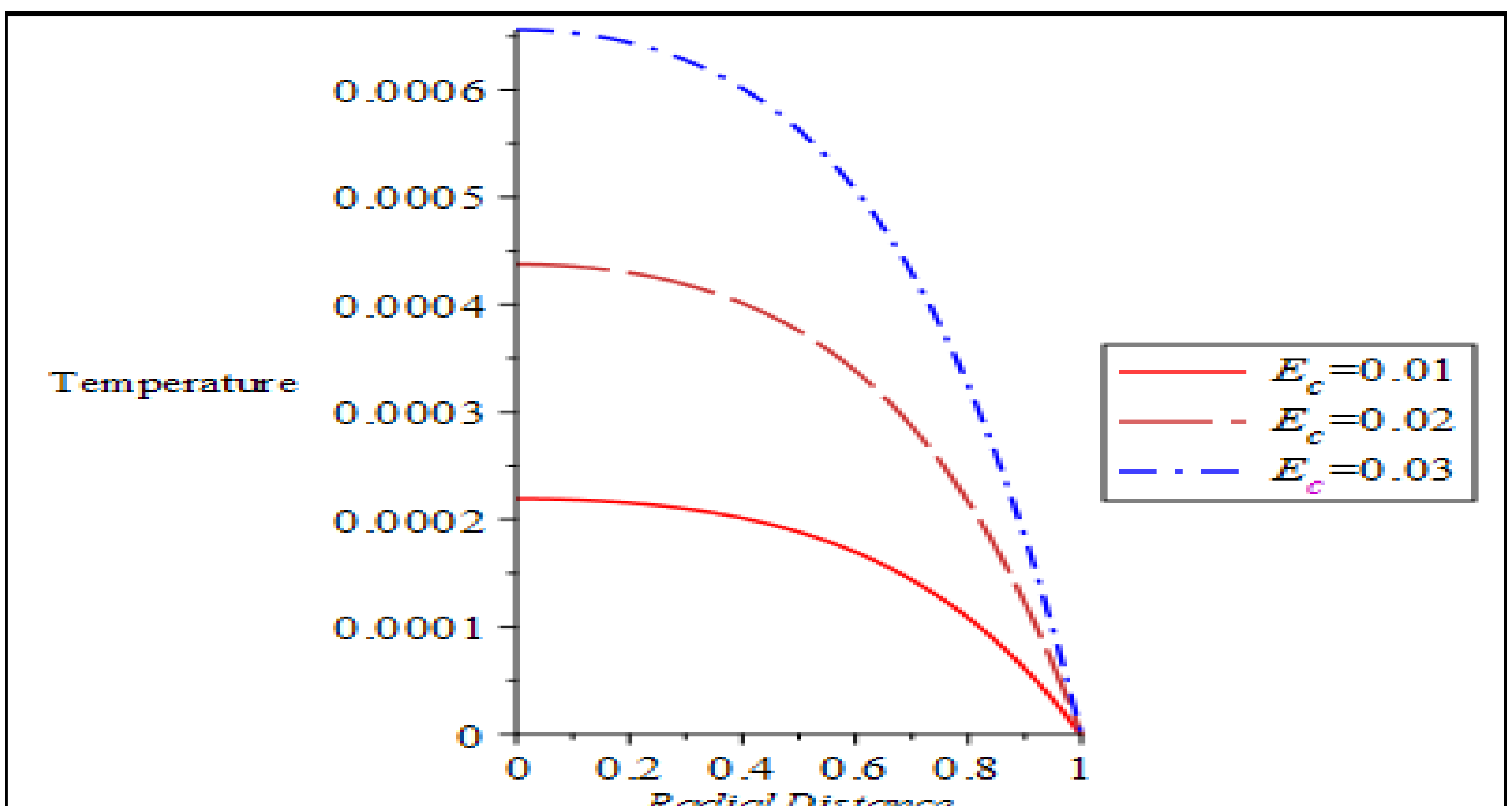

Figure 5: Temperature Profiles For Various Values of The Eckert Number (E_c)

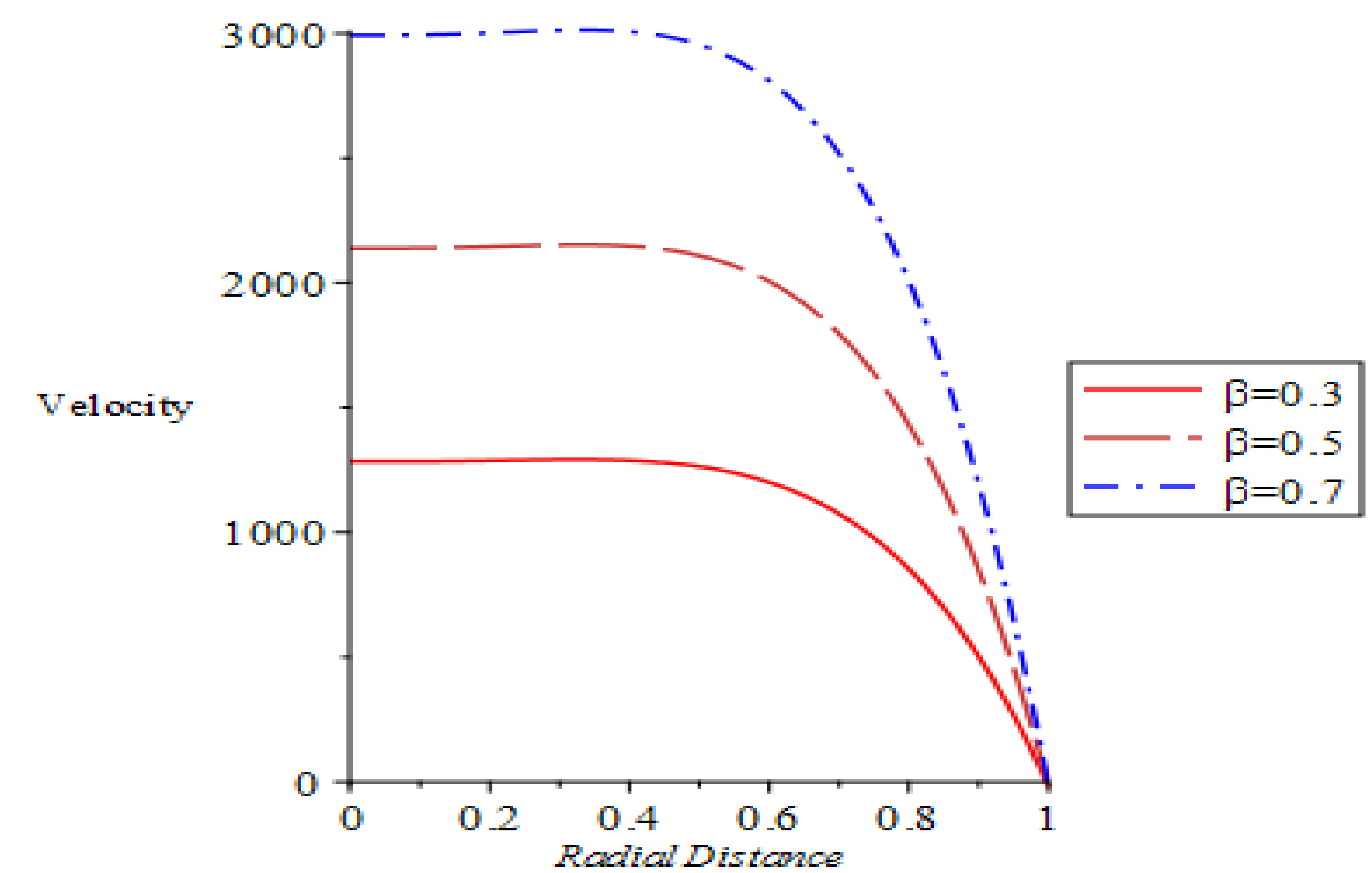

Figure 6: Velocity Profiles For Various Values of Vogel Viscosity Index (β)

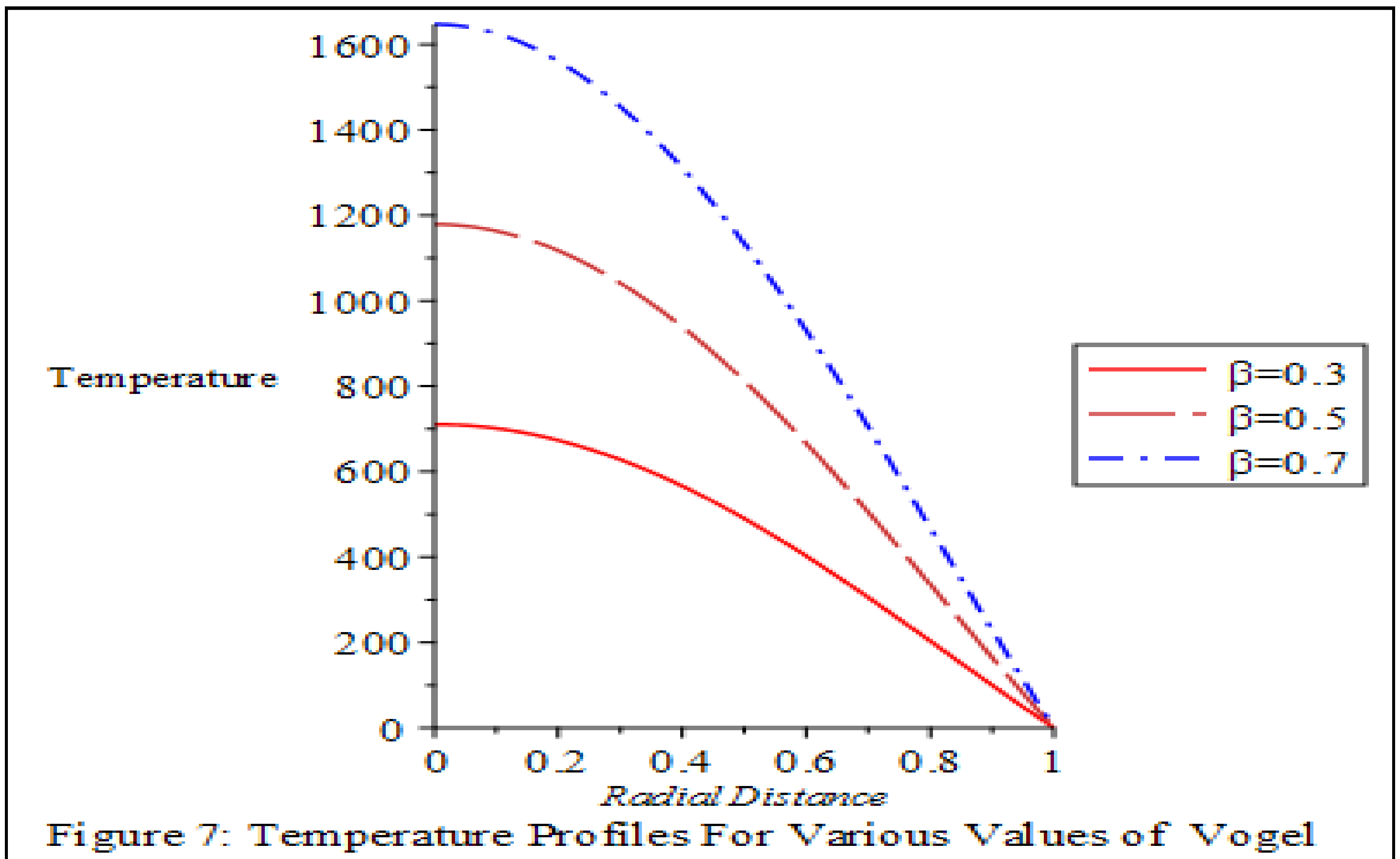

Figure 7: Temperature Profiles For Various Values of Vogel Viscosity Index (β)

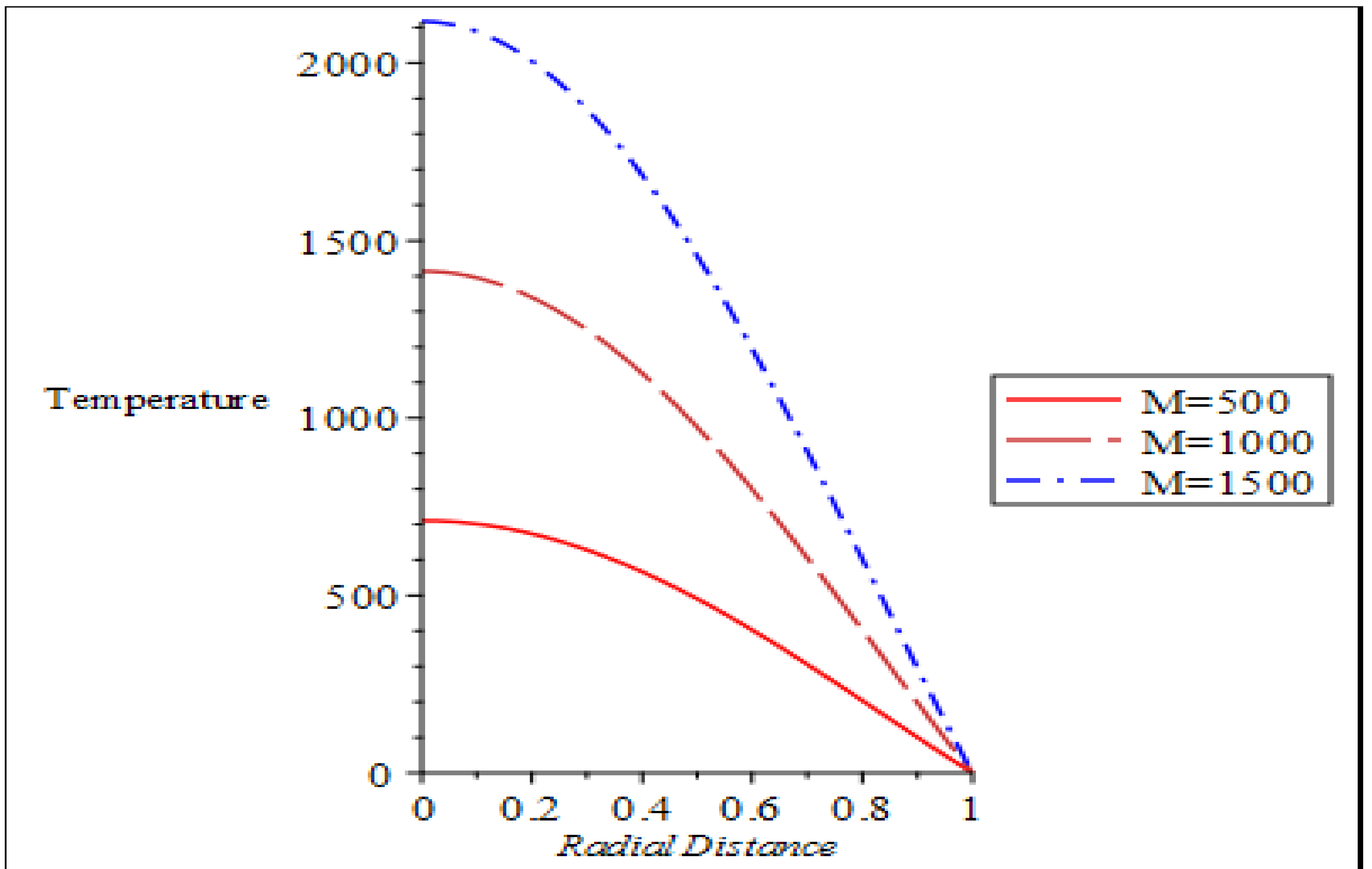

Figure 8: Temperature Profiles For Various Values of Vogel Viscosity Index (M)

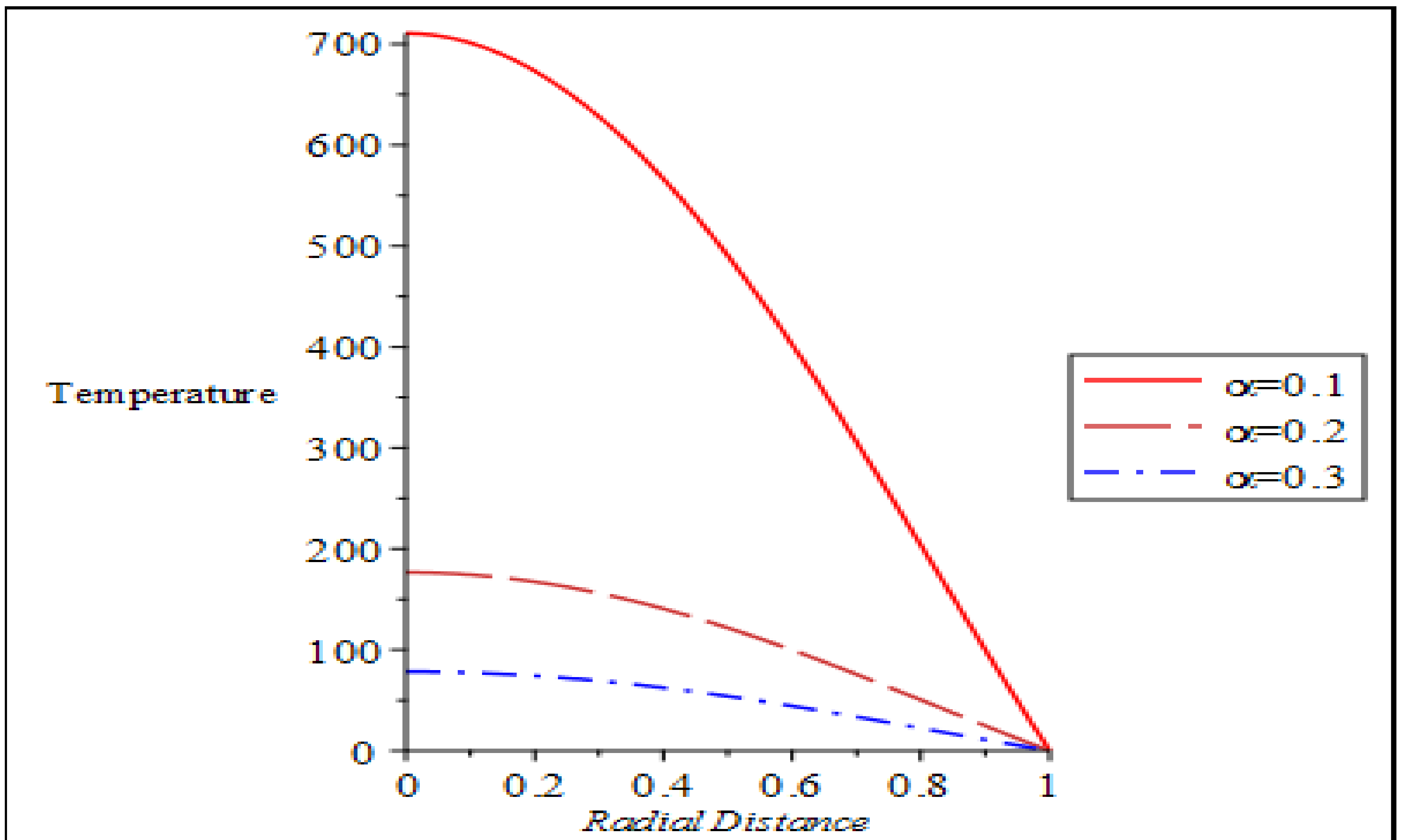

Figure 9: Temperature Profiles For Various Values of Vogel Viscosity Index (α)

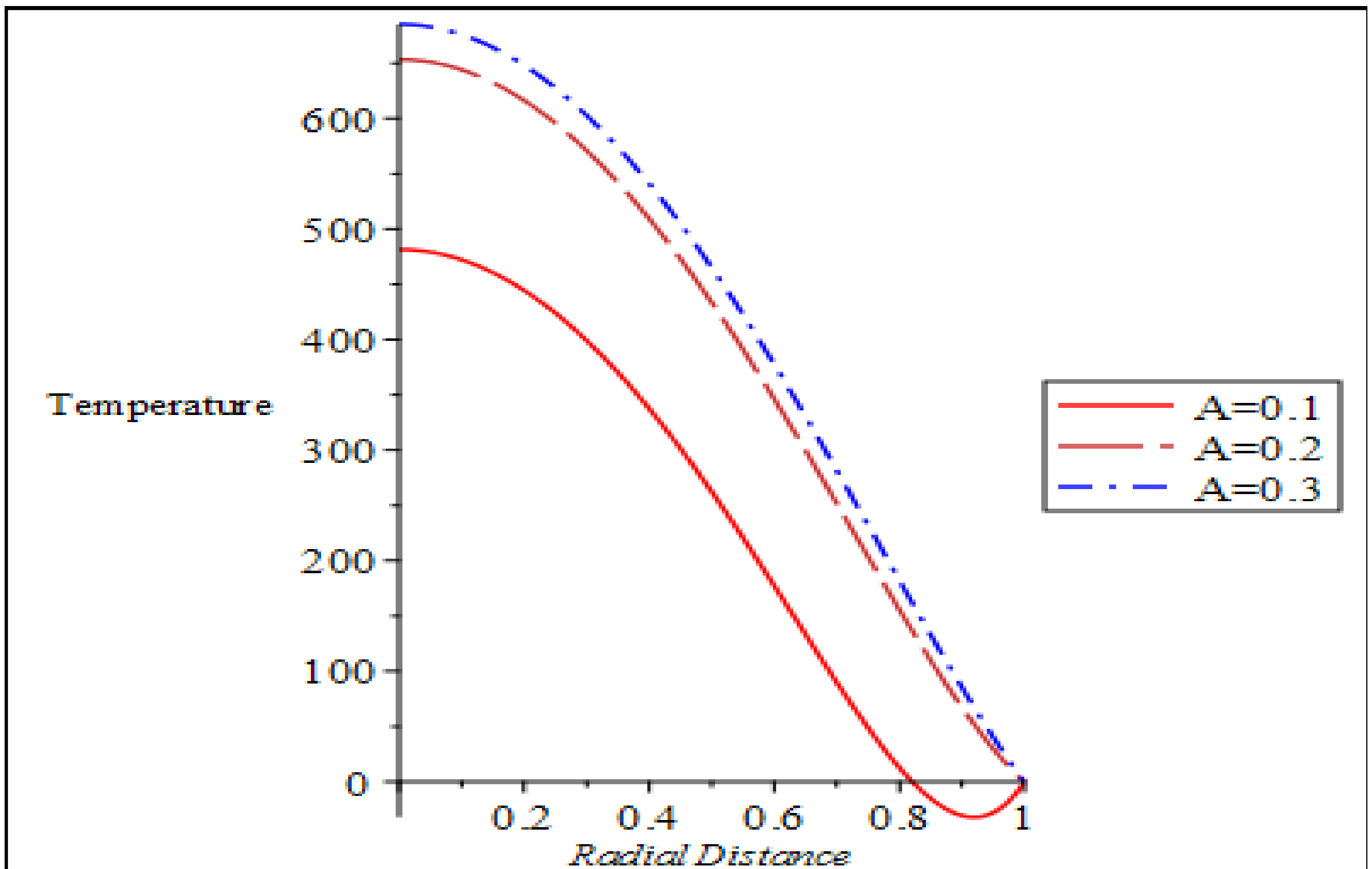

Figure 10: Temperature Profiles For Various Values of Vogel Viscosity Index (A)

4. Results and Discussion

In order to study the behaviour of some physical parameters involved in the analysis, graphs are presented in Figures 1 to 10. The solution of momentum and energy equations (1) and (2) with the boundary condion in (3) given in (13) and (14). Figure 1 shows the effects of third grade parameter on the velocity profiles. Results indicate that increase in third grade parameter decreases the flow velocity. This is because the third grade parameter introduces a relationship between the velocity and the radial distance from the boundary which is nonlinear and leads to drop in flow velocity. Figure 2 is the velocity profiles for various values of the magnetic field. It is observed from the results that increase in the magnetic field decreases the velocity because the applied force set in by the magnetic field parameter is perpendicular to the flow direction. Figure 3 shows temperature profiles for different values of third grade parameter. Results show that as the third grade parameter increases, the temperature of the cylindricxal pipe increases as well owning to the thermal conductivity of the fluid which influences the temperature distribution. It is observed in Figures 4 and 5 that the magnetic field and Eckert parameters increases the temperature when the parameters are increased at a regular rate. Reasults further show that the Eckert parameter regulates the rate of heat transfer. Figures 6 and 7 show the velocity and temperature profiles for different values of the the third grade parameter within the Vogel model analysis respectively. Results show that as the third grade parameter increases, both the velocity and temperature increases at the same rate. In Figure 8, the temperature profiles for various values of the the magnetic field parameter is shown. It is seen that increase in the magnetic field parameter increases the temperature at the walls of the cylinder. Figure 9 shows the temperature profiles for different values of the Vogel parameter α . It is observed that the parameter has the propensity to lower the temperature when increased at very small rate. Figure 10 shows the second major parameter, A, in the Vogel model structure. Results indicate that increase in the parameter, A, increases the temperature of the cylindrical walls.

5. Conclusions

Analytical study of incompressible MHD non-Newtonian fluid in cylindrical pipe with isothermal wall and temperature-dependent viscosity is examined. Vogel's model viscosity is introduced to account for the temperature-dependent viscosity, while the third grade fluid is accommodated to model the non-Newtonian fluid feature. It is observed that the third grade and the magnetic field parameters reduces the velocity profiles and increases the temperature profiles when increased at a steady rate within the constant viscosity index but increases the velocity and the temperature profiles when subjected to the Vogel model. Meanwhile the Eckert parameter is observed to enhance the temperature near the walls of cylindrical pipe. Results further show that increasing the two Vogel model indices, α and A , decreases and increases the temperature profiles, respectively.

6. References

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